



National Aviation University of Ukraine
Air Navigation Systems Department

Risk Analysis of Positioning by Navigational Aids

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Ostroumov Ivan, Ph.D.
Associate professor of
Air Navigation System Department



Kuzmenko Nataliia, Ph.D.
Senior researcher

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Aircraft on-board equipment for positioning

Global Navigation Satellite System
(GPS, GLONASS, GALILEO, BeiDou)

Inertial Navigation System
(IRS, AHRS)

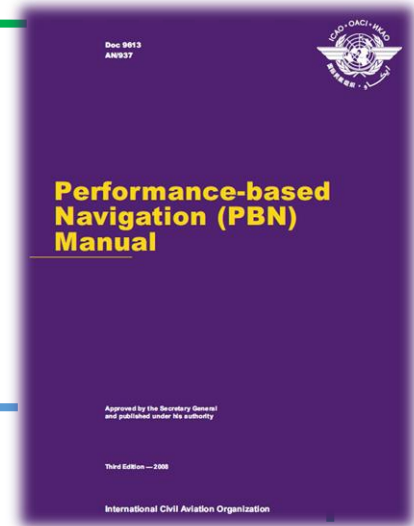
Position detection by pair of DME (DME/DME)

Position detection by pair of VOR and DME (VOR/DME)

Positioning by Navigational Aids in Flight Management System

Compatible
with PBN
(RNAV 1)

ICAO
DOC 9613

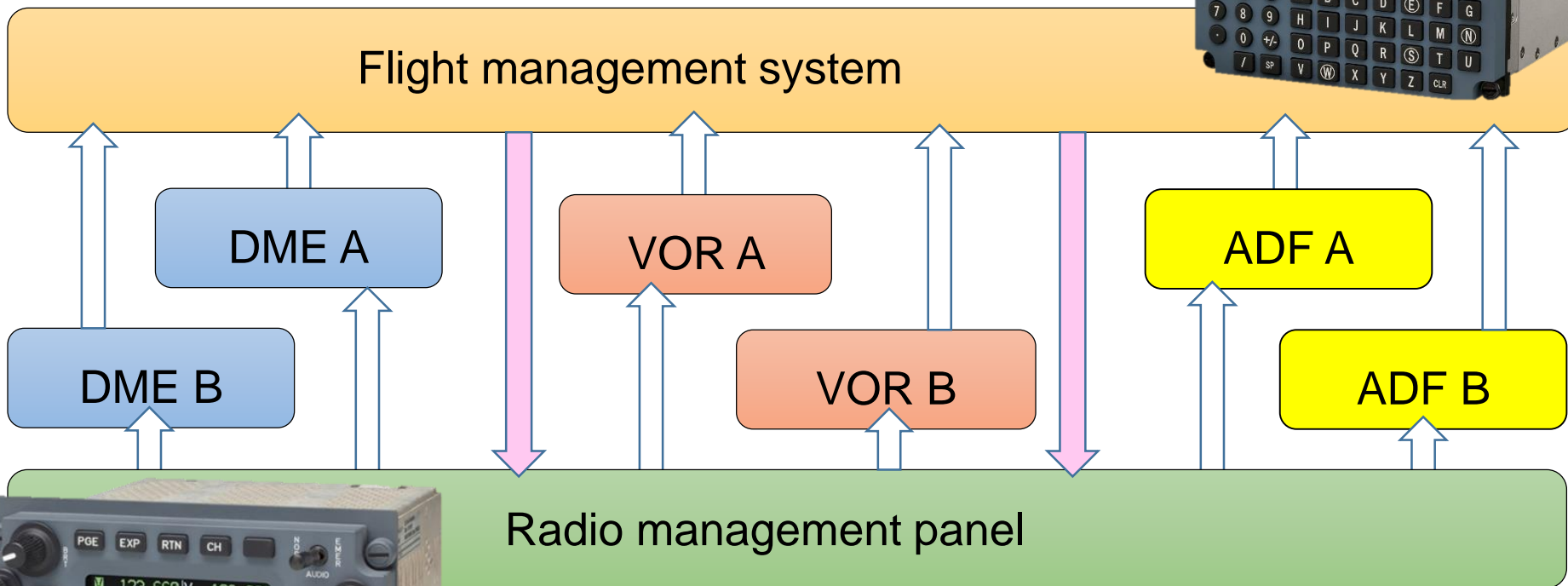


Errors of Global Navigation Satellite Systems

- large spatial variation in ionospheric delay
- **Interference from ground based radio equipment**
- **jamming of radio waves**
- Refraction from obstacles (multipath error)
- Geometric factor of the satellite segment



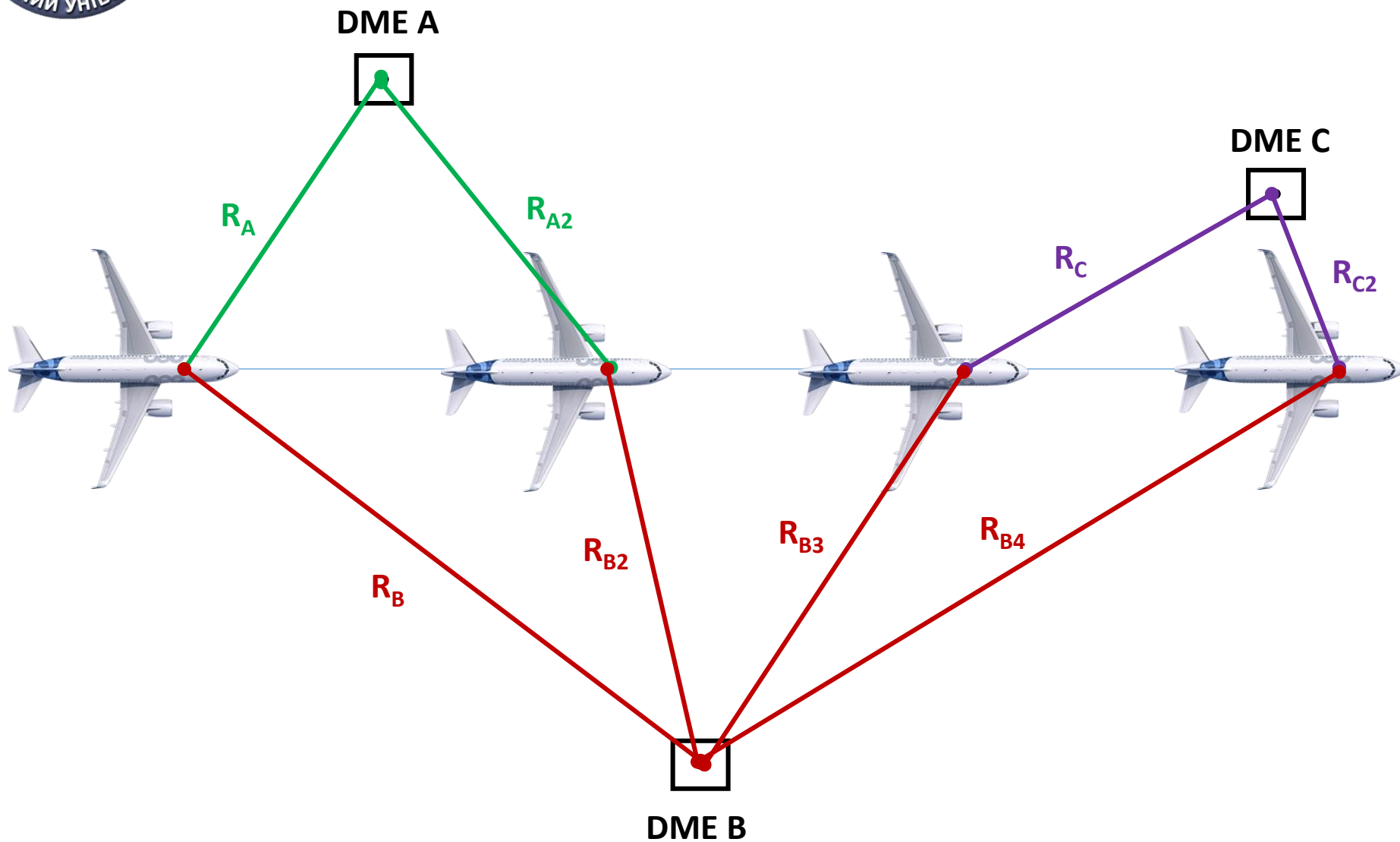
Interchange of navigational aids data of on-board equipment



Only pair of DME/DME equipment is in use for positioning function.



DME/DME





Error of positioning in DME/DME

$$\sigma_P = \frac{\sqrt{\sigma_{DMEA}^2 + \sigma_{DMEB}^2}}{\sin(\alpha_{AB})}$$

$$\sigma_{DMEA,B}^2 = \sigma_{sis}^2 + \sigma_{air}^2$$

σ_{sis}^2 – Signal in space error

σ_{air}^2 – Error of airborne interrogator

$$\sigma_{sis} = 0.05 \text{ NM}$$

RTCA DO-189:

$$\sigma_{air} = 0.085 \text{ NM}$$

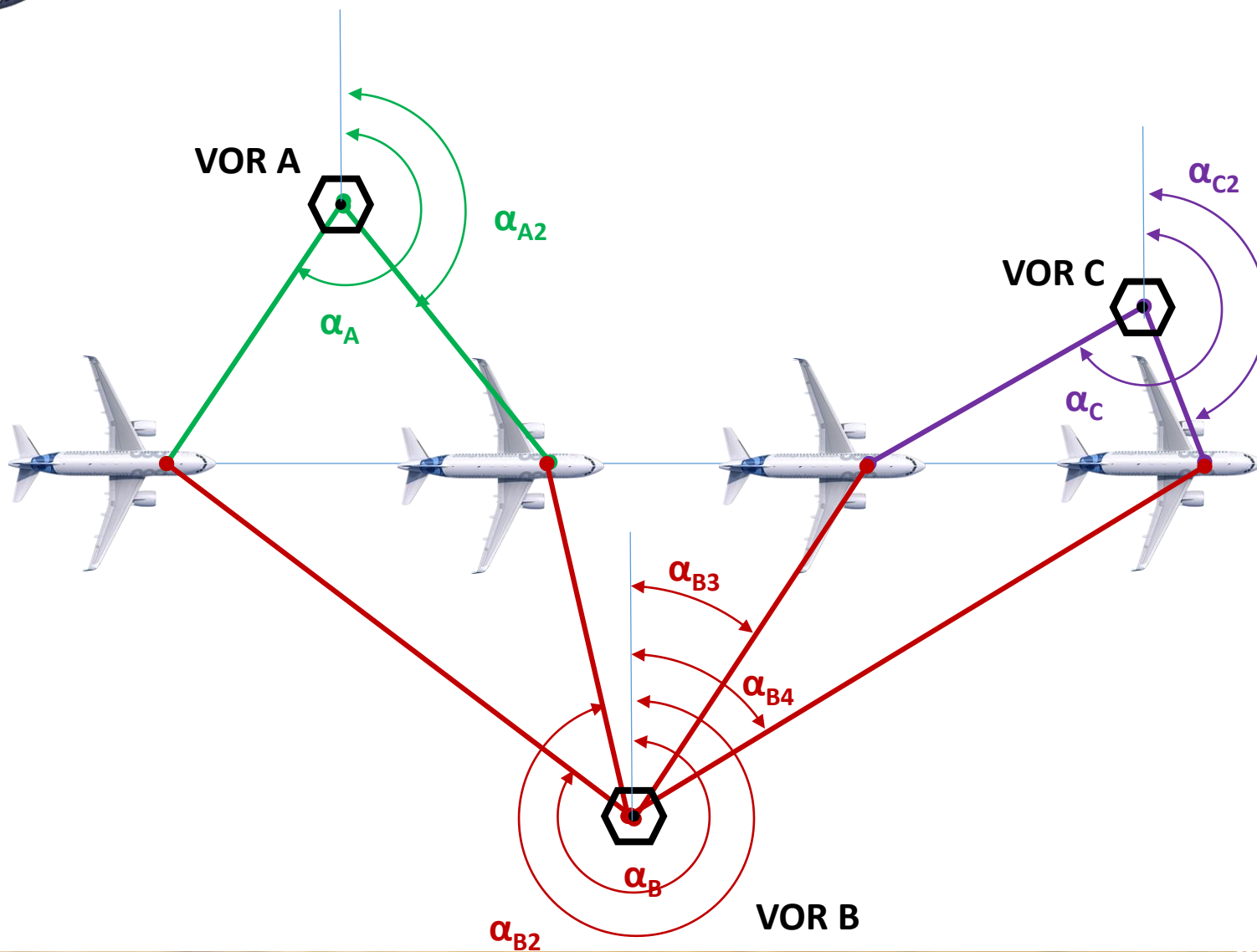
AC90-100A and ICAO DOC-9613:

$$\sigma_{air} = \max\{0.085 \text{ NM}; 0,125\% R\},$$

$$\sigma_P = \frac{\sqrt{2\sigma_{sis}^2 + \sigma_{airA}^2 + \sigma_{airB}^2}}{\sin(\alpha_{AB})}$$

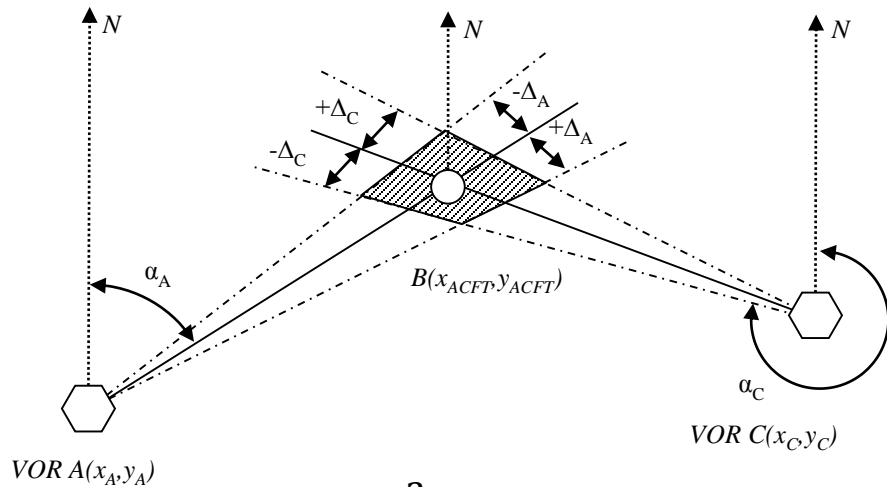


VOR/VOR





Error of positioning in VOR/VOR

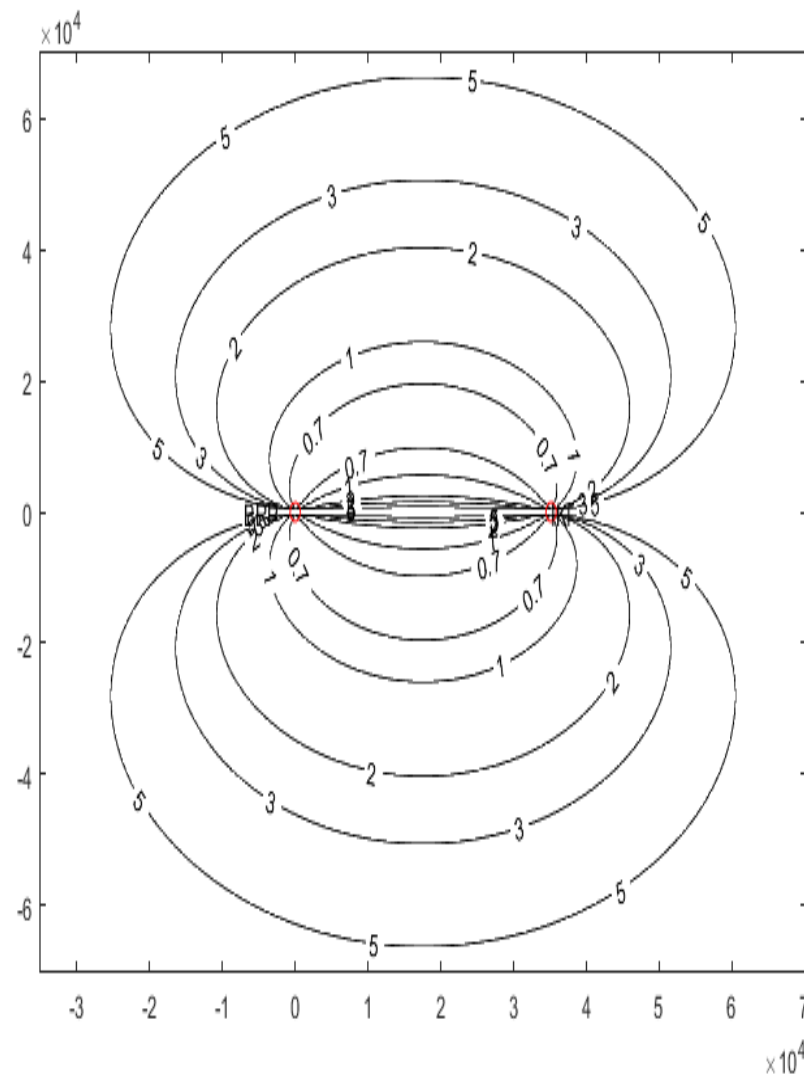
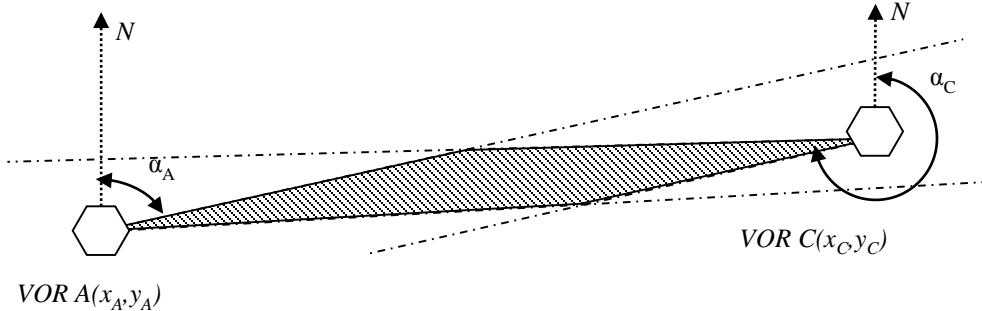


$$\sigma_p^2 = \frac{\sigma_\alpha^2}{\sin^2(\alpha)} (d_A^2 + d_B^2)$$

Where:

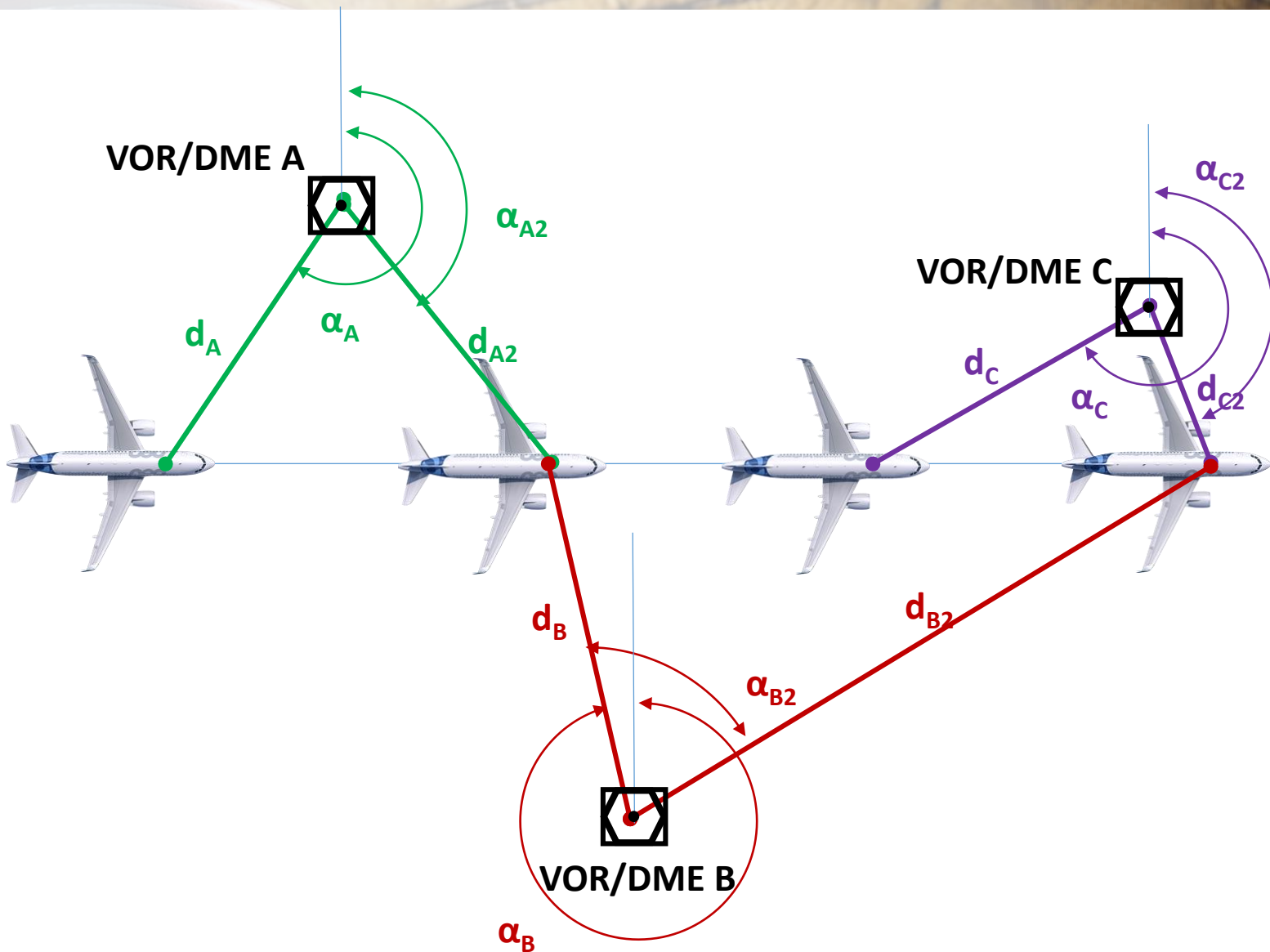
$$d_A = \frac{d \sin(\alpha_B)}{\sin(\alpha_A + \alpha_B)}$$

$$d_B = \frac{d \sin(\alpha_A)}{\sin(\alpha_A + \alpha_B)}$$



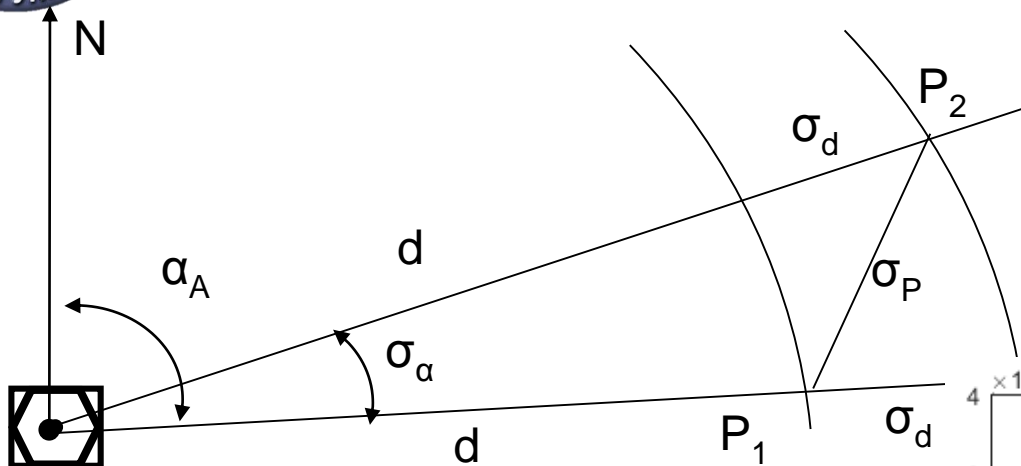


VOR/DME



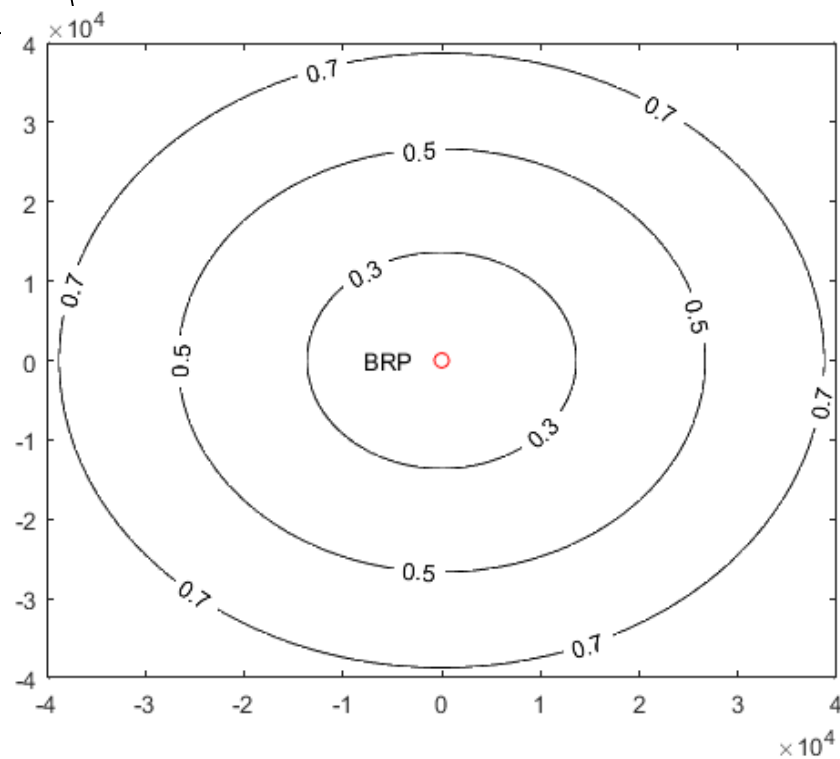


Error of positioning in VOR/DME



VOR/DME

$$\sigma_p^2 = \sigma_{DME}^2 + D^2 \sigma_{VOR}^2$$





Optimization problem

NAVAIDs optimal pair selection is a typical optimization problem which can be represented as an objective function and constraints in terms of integer linear programming

Objective function: $WX^T \rightarrow \min$,

Constraints: $AX^T \geq Nmax$

- by NAVAIDs availability in space

$$GX^T \geq 30^\circ$$

- by inclusion angle

$$-GX^T \geq -150^\circ$$

$$IX^T = 1$$

- by number of solution

$$X \geq 0$$

$$X \in \{0, 1\}$$

where

$X = [x_1, x_2, x_3, \dots, x_n]$ – binary matrix which includes indices of pairs;

$W = [W_{DME/DME}, W_{VOR/VOR}, W_{VOR/DME}]$ – weight matrix of each pair of NAVAIDs;

– for DME/DME: $W_{DME/DME} = [\sigma_{DME/DME 1}, \sigma_{DME/DME 2}, \sigma_{DME/DME 3}, \dots, \sigma_{DME/DME n}]$;

– for VOR/VOR: $W_{VOR/VOR} = [\sigma_{VOR/VOR 1}, \sigma_{VOR/VOR 2}, \sigma_{VOR/VOR 3}, \dots, \sigma_{VOR/VOR n}]$;

– for VOR/DME: $W_{VOR/DME} = [\sigma_{VOR/DME 1}, \sigma_{VOR/DME 2}, \sigma_{VOR/DME 3}, \dots, \sigma_{VOR/DME n}]$;

$A = [a_1, a_2, a_3, \dots, a_n]$ – availability binary matrix;

$G = [g_1, g_2, g_3, \dots, g_n]$ – matrix of inclusion angles.



Risk assessment model

A bivariate normal distribution density function (BNDDF) is used as an error model

$$\rho(x) = \frac{1}{2\pi|S|^{-1/2}} \exp\left[-\frac{1}{2}(S_k^{-1}(x-M)^T \times (x-M))\right]$$

where x are measurements; $M=\{\mu_1, \mu_2\}$ are coordinates of the aircraft location in a certain Cartesian system; S is diagonal matrix of mean-square deviations:

$$S = \begin{vmatrix} TSE_1^2 & 0 \\ 0 & TSE_2^2 \end{vmatrix}$$

A risk of navigation system lock in case of BNDDF can be visualized as a volume of a figure limited by density function and a cylinder, which set up limits of area navigation:

$$R_{pos} = 1 - \iint_{2TSE_{RNAV}} \rho(x) dx$$

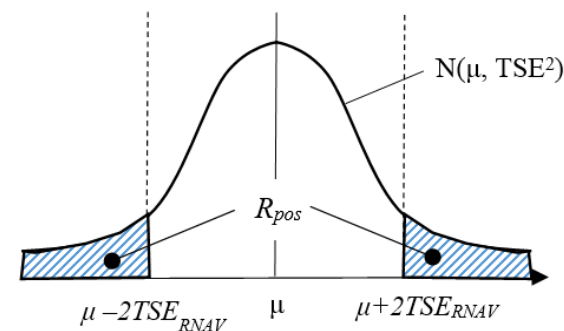


Fig. Error density function

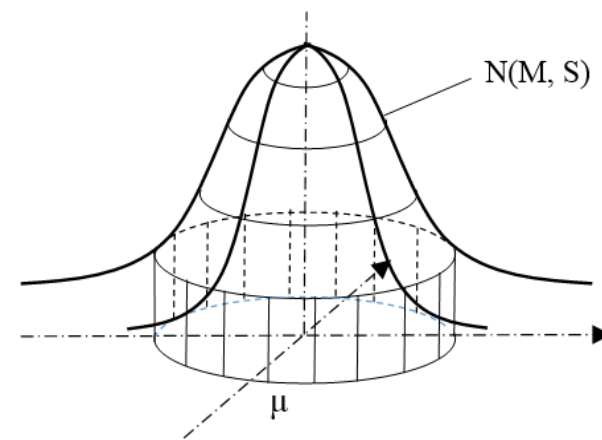


Fig. Risk assessment in case of a bivariate model



Standard Service Volumes

Ground stations classification

T (Terminal)

From 1000 feet (305 m) AGL up to and including 12,000 feet (3,658 m) AGL at radial distances out to 25 nm (46 km).

L (Low Altitude)

From 1000 feet (305 m) AGL up to and including 18,000 feet (5,486 m) AGL at radial distances out to 40 nm (74 km).

H (High Altitude)

From 1000 feet (305 m) AGL up to and including 14,500 feet (4,420 m) AGL at radial distances out to 40 nm (74 km).

From 14,500 feet (4,420 m) AGL up to and including 60,000 feet (18,288 m) at radial distances out to 100 nm (185 km).

From 18,000 feet (5,486 m) AGL up to and including 45,000 feet (13,716 m) at radial distances out to 130 nm (241 km).

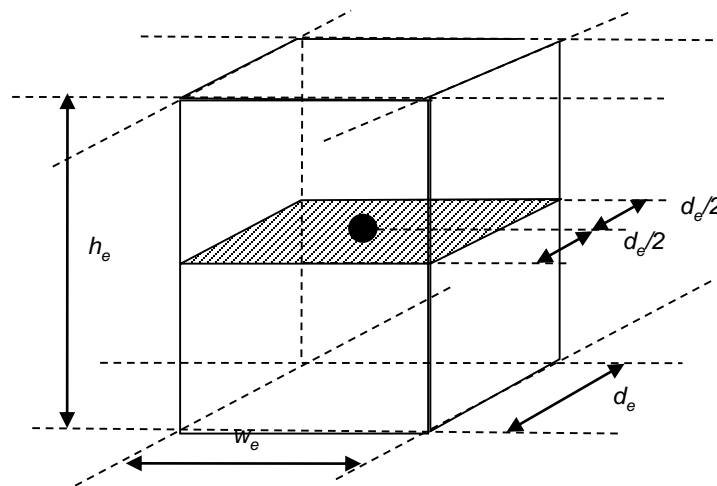
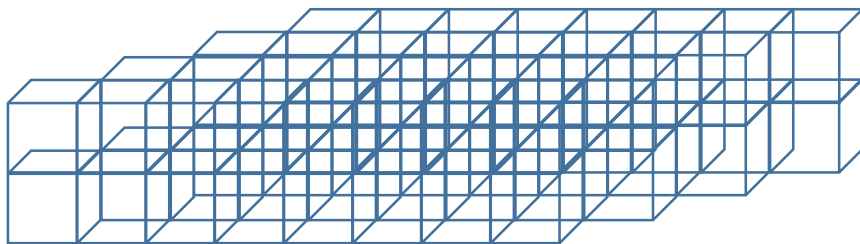
AGL – Above Ground Level

FAA AC 00-31A



Performance estimation

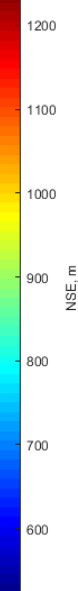
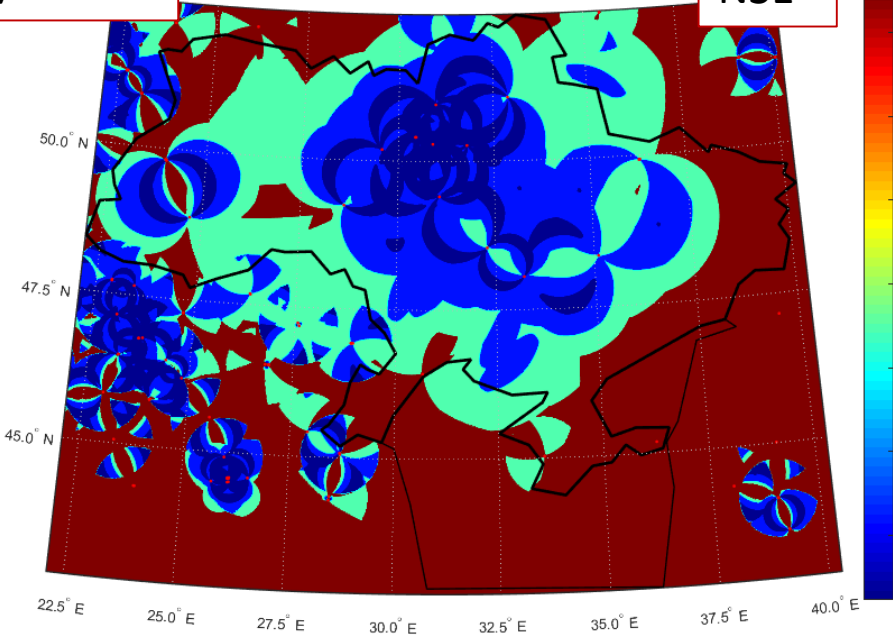
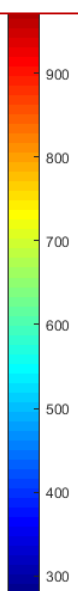
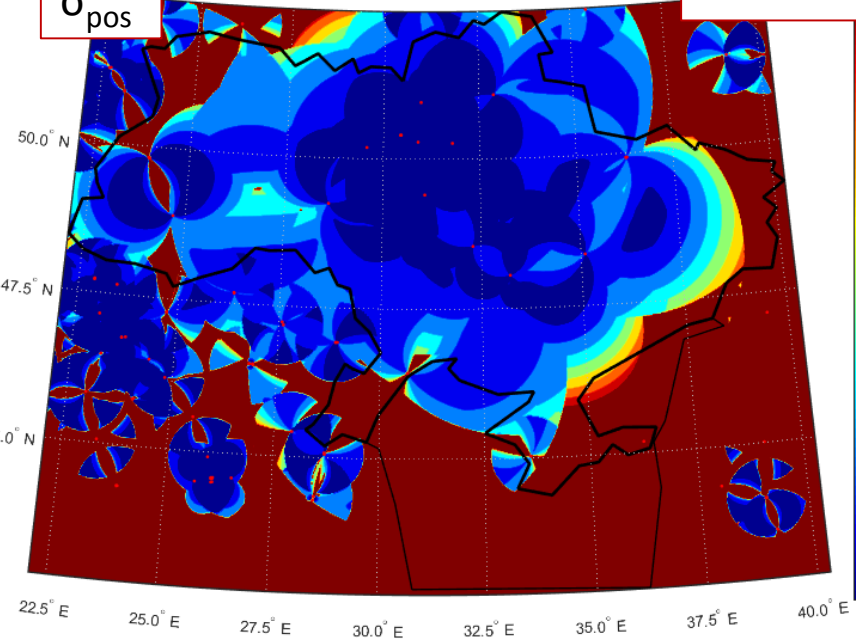
1. Iterative approach with dividing air space into elementary cells
2. Area of investigated airspace is 853412,1 k km²
3. Models of NAVAIDS service volume estimation
 - Cylindrical models by AC 00-31A, + approach, + en-route
 - Radio waves propagation; for antenna “dbs5100” DME; relief model included
 - DEM: SRTM (February 11-22, 2000) 3 arc sec (possible 1 arc sec)
4. Optimal pair of DME/DME, VOR/DME, and VOR/VOR respectively for the FL 290 altitude



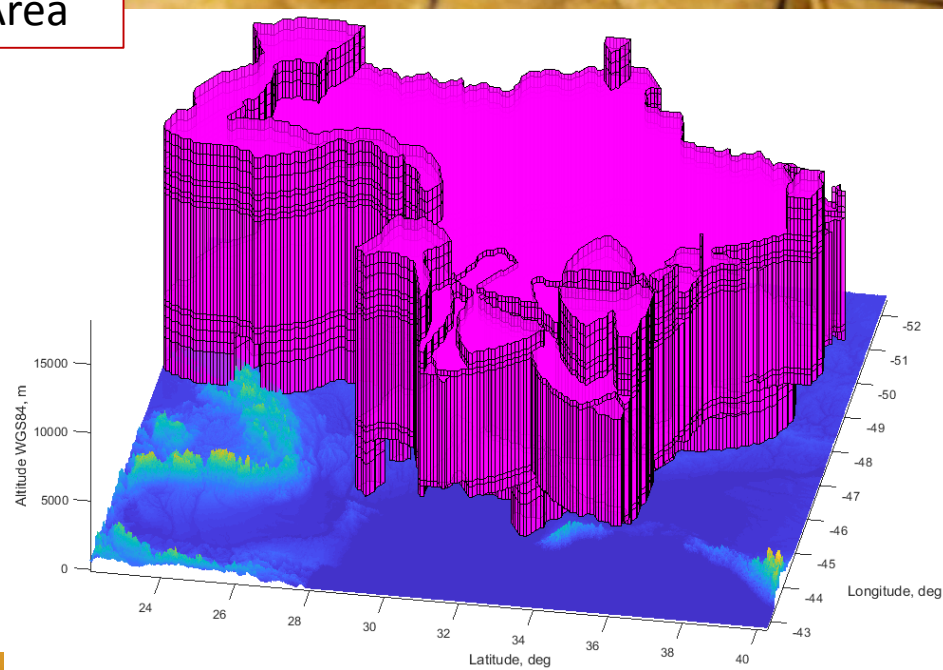
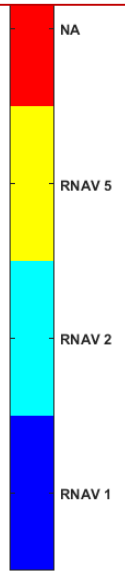
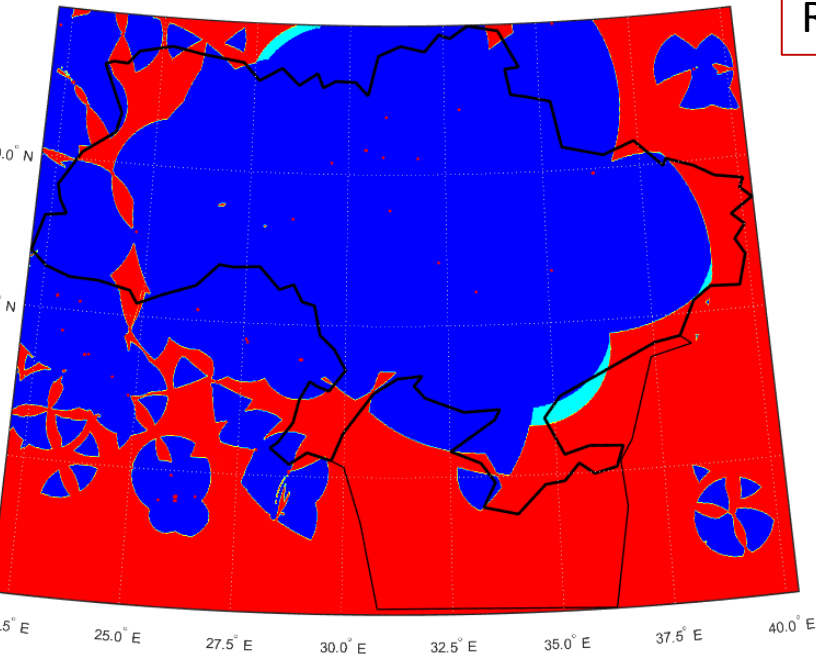
FL 105 DME/DME

σ_{pos}

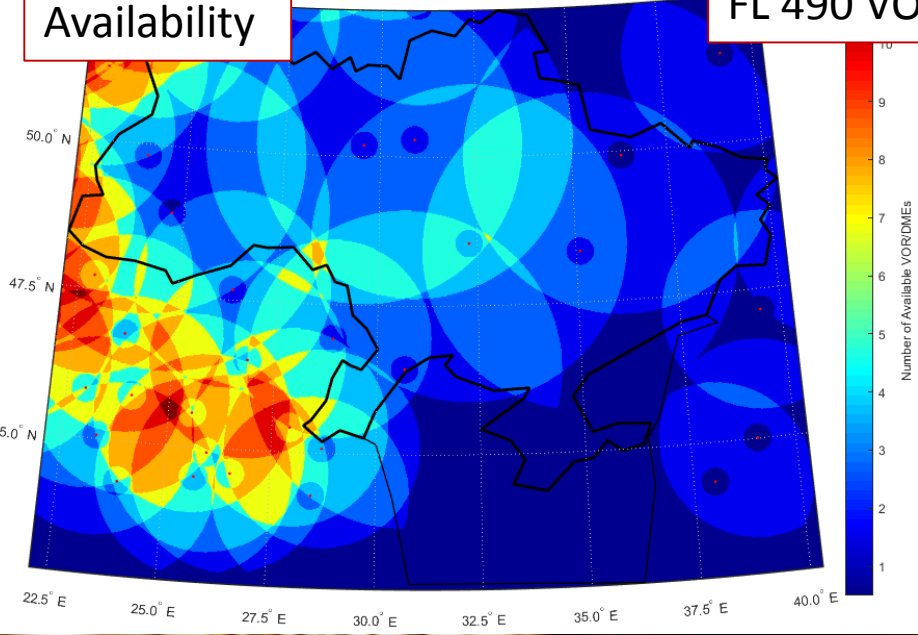
NSE



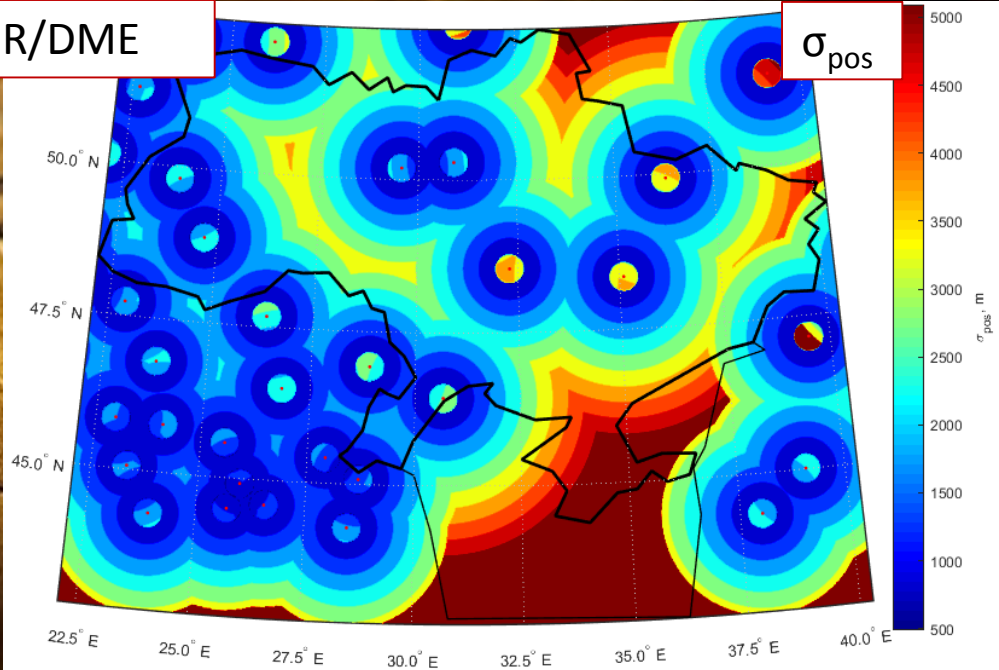
RNAV1 Area



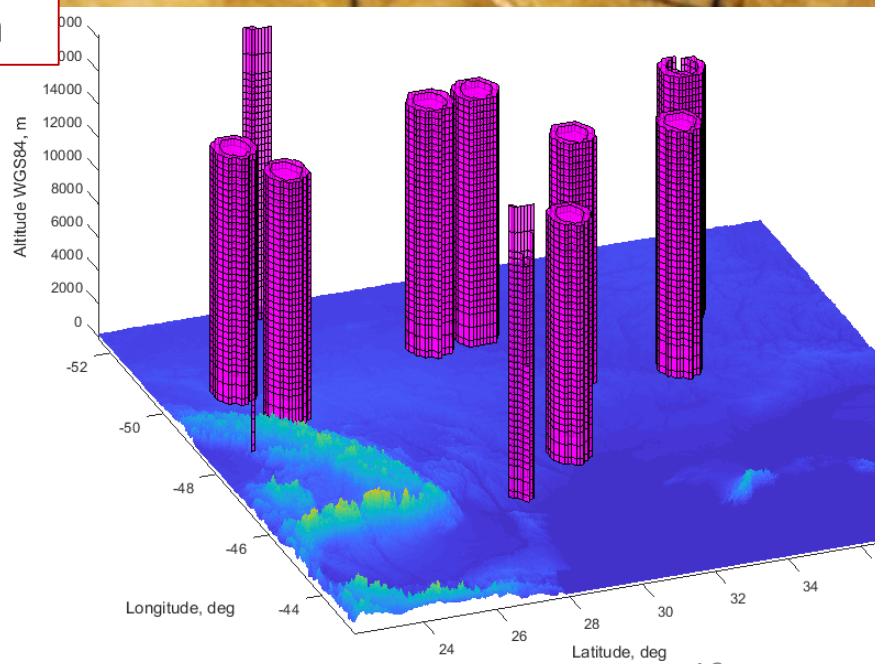
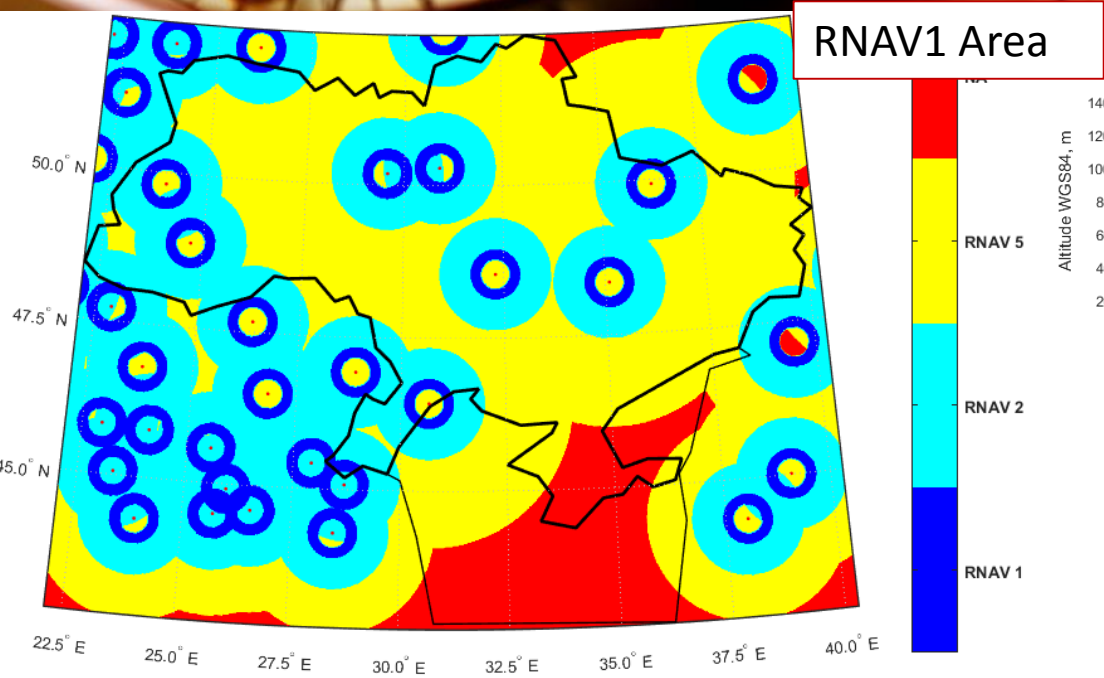
Availability



FL 490 VOR/DME

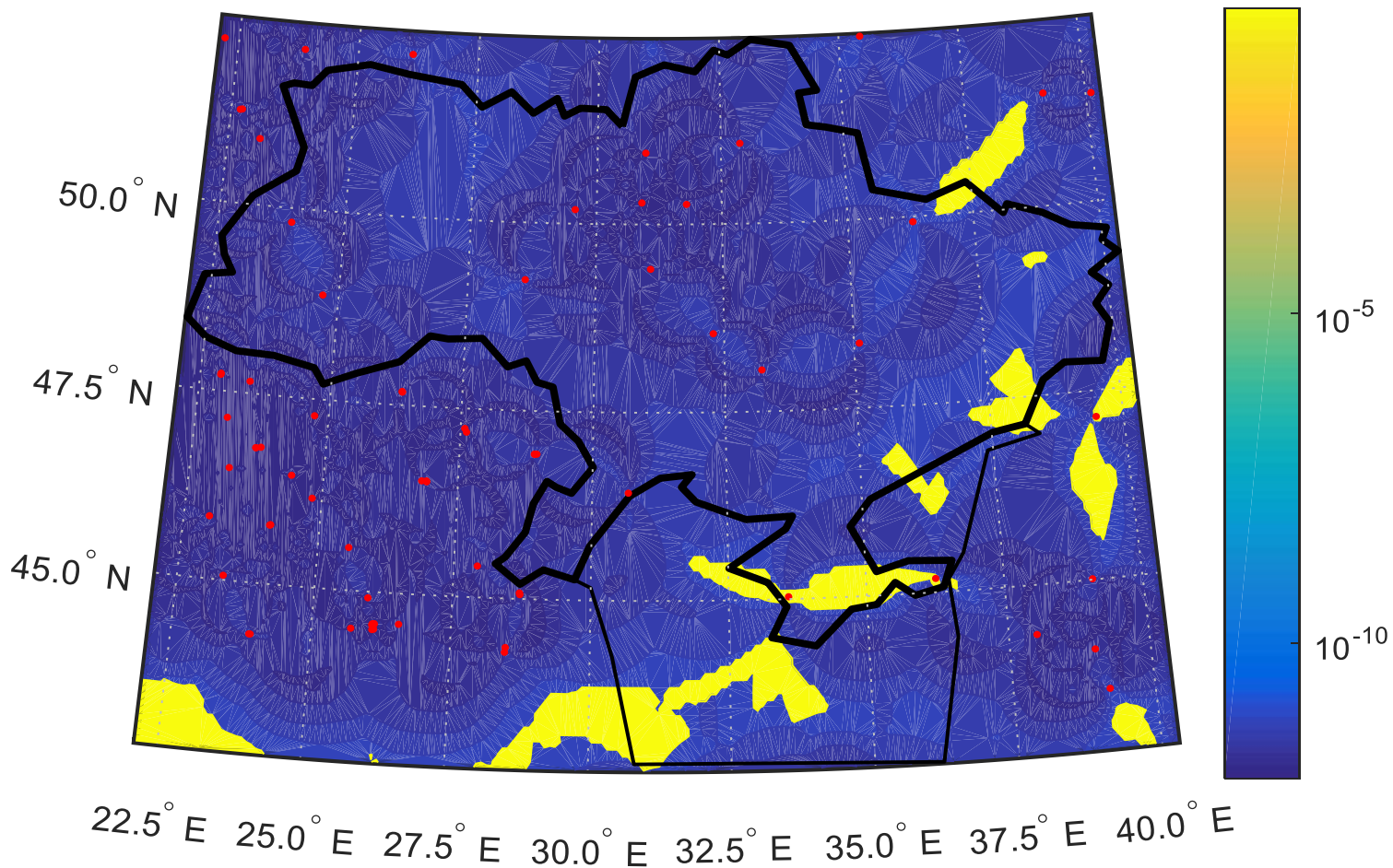


RNAV1 Area



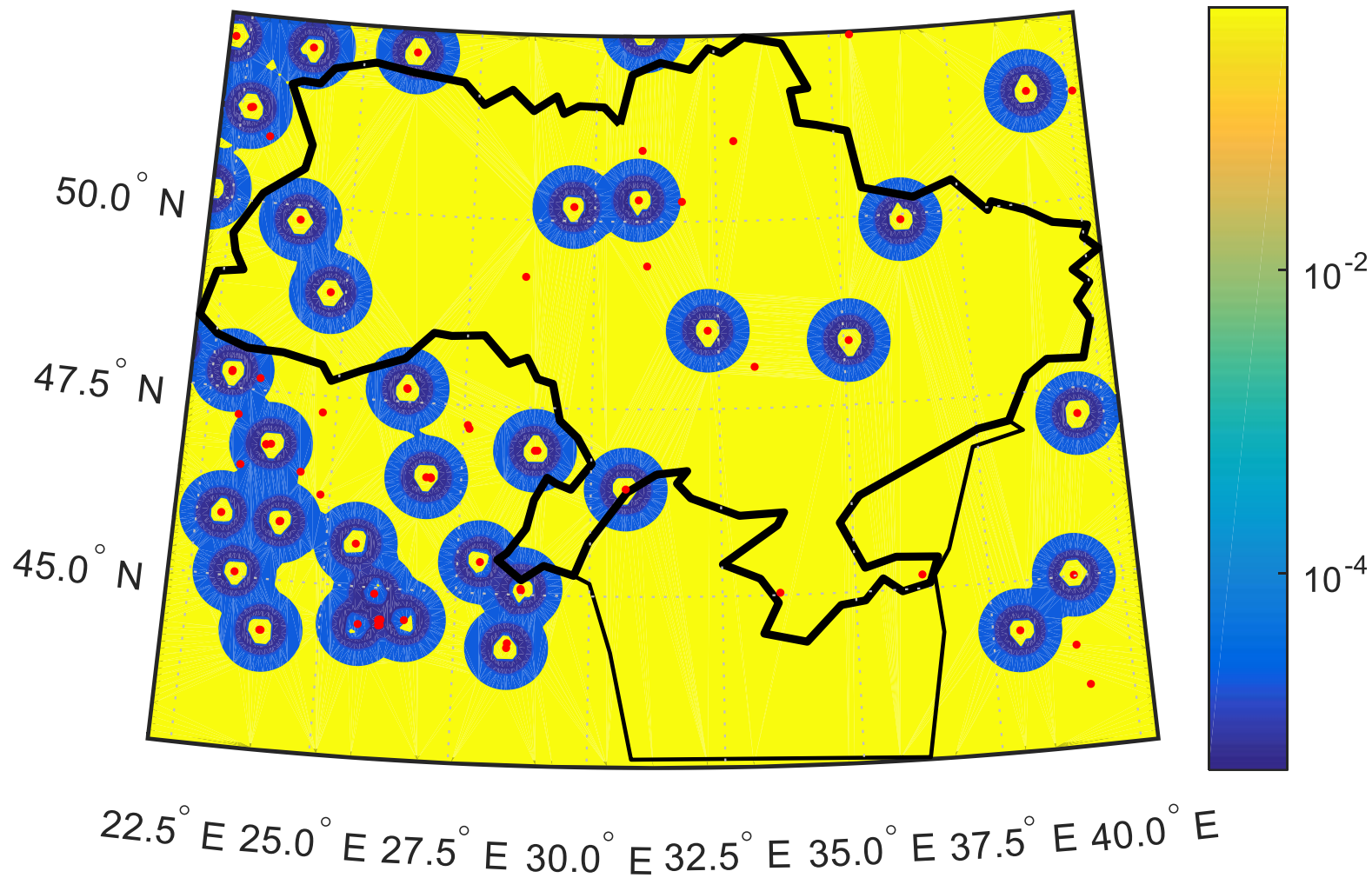


Risk for DME/DME positioning algorithm



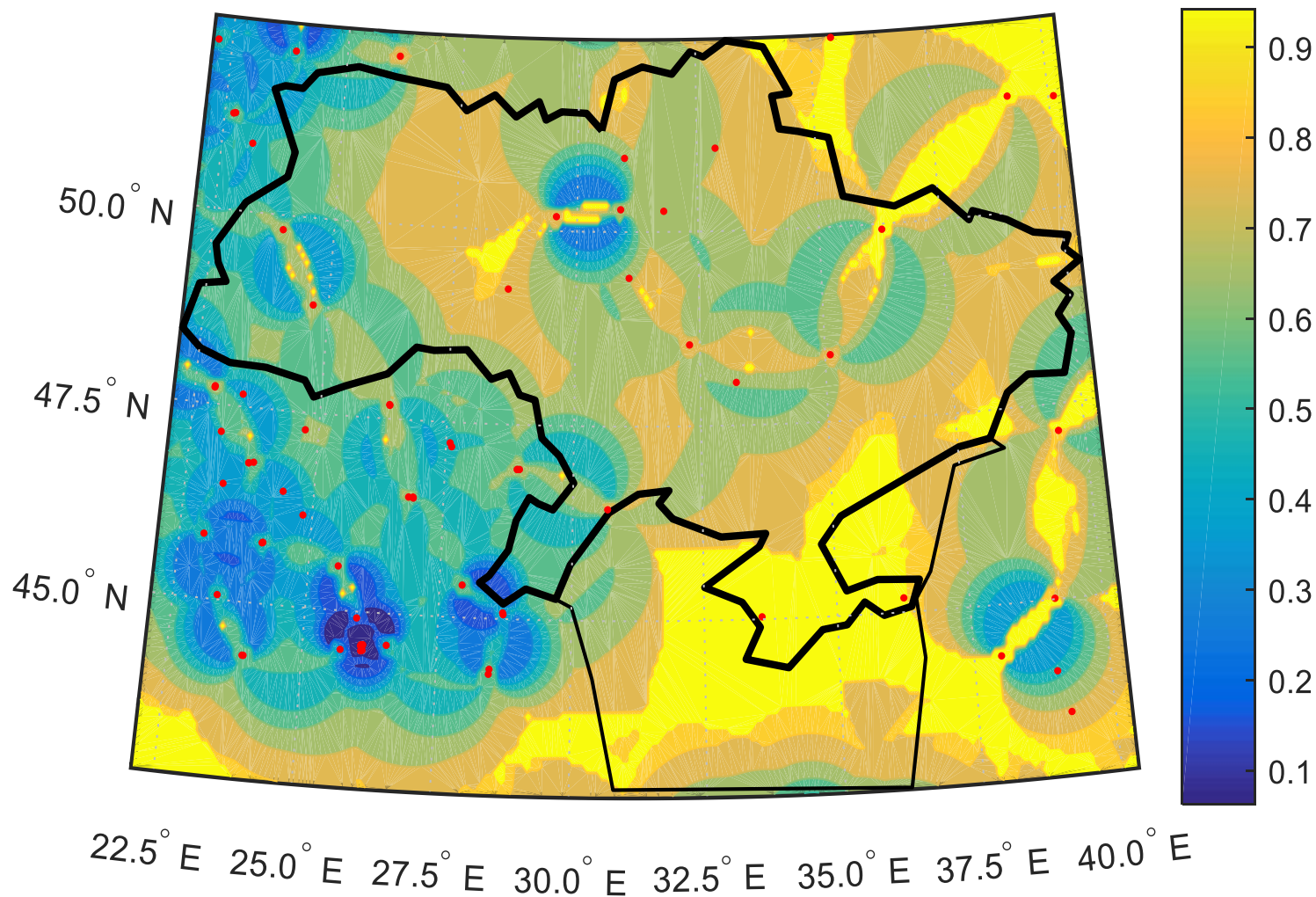


Risk for VOR/DME positioning algorithm



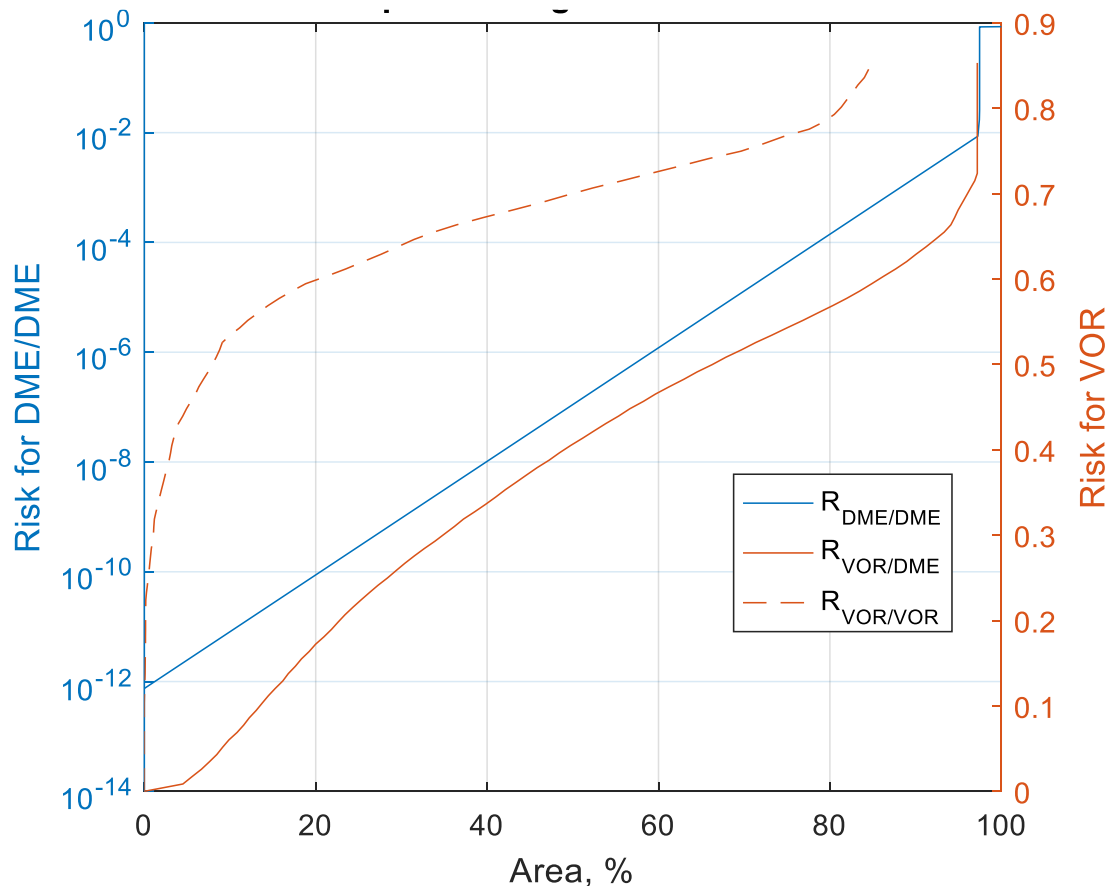


Risk for VOR/VOR positioning algorithm





An area of positioning lock by different positioning systems



The lowest risk value can be obtained for positioning by optimal DME/DME pair. In particular, 80% of the Ukrainian airspace at FL 290 has the risk level of positioning by DME/DME less than 10^{-4} , 60% has 10^{-6} , 40% has 10^{-8} , and 20% has only 10^{-10} . However, VOR/DME and VOR/VOR positioning methods do not support risk levels below 10^{-2} .



Conclusions

The results of simulation indicate possibility of applying the risk of positioning system lock as a probabilistic indicator of compliance with the air navigation specification requirements. Lower risk values correspond to more precise TSE values. Result of risk analysis of positioning system lock within the Ukrainian airspace indicates low risk for positioning by optimal DME/DME pair, that is result of rational configuration of the DME ground network, while the use of VOR for positioning increases the risk level more than 50% of investigated airspace. High risk values for VOR/VOR and VOR/DME are result of the quite pure ground network of VOR equipment.

The proposed risk of positioning system lock is an important value for the theory of aviation risk analysis in cases of aircraft deviation from the given trajectory, various aircraft collision analysis and should be taken into account during the airspace safety analysis.

Contacts

www.ostroumov.sciary.com

ostroumovv@ukr.net