



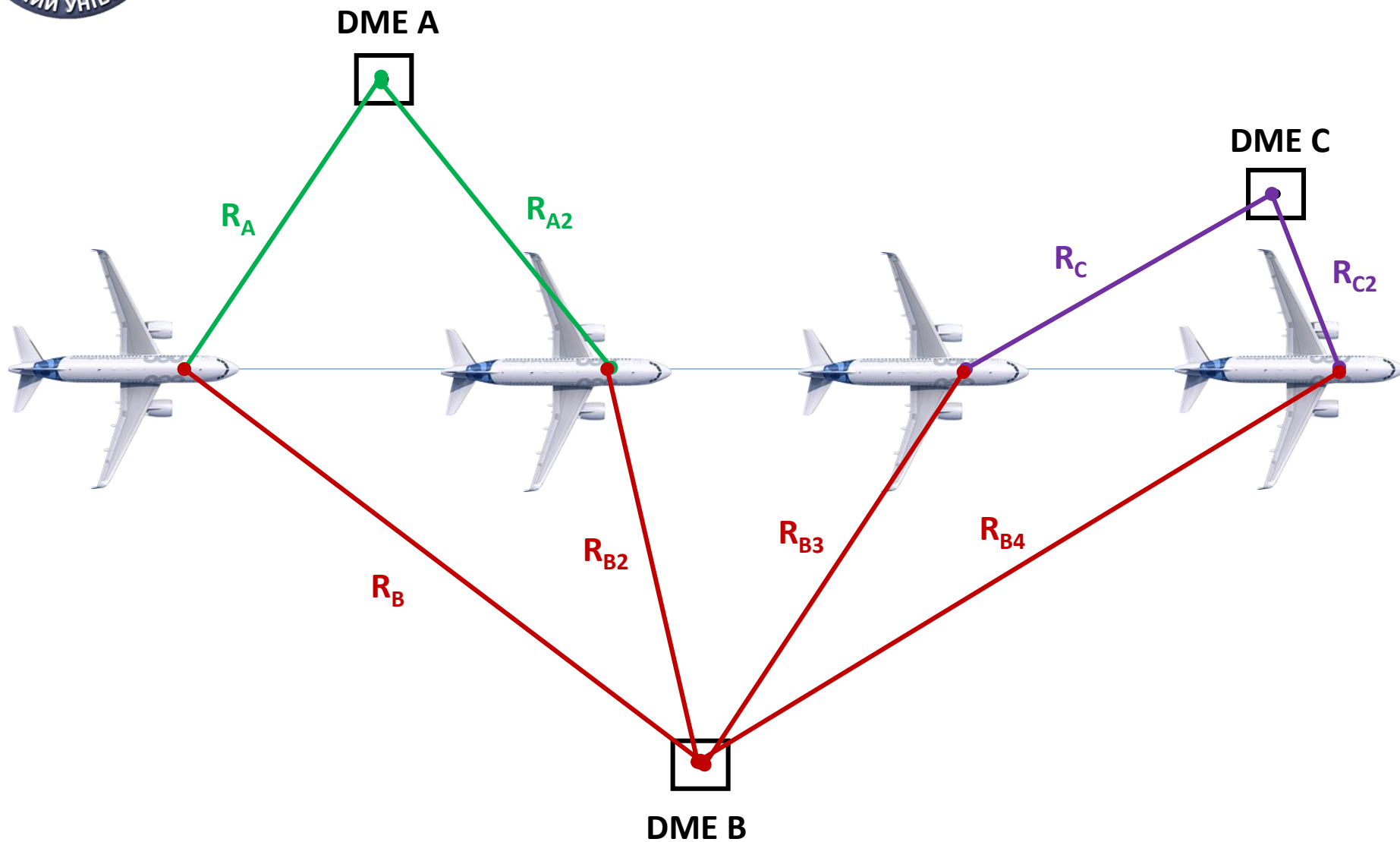
Improving the Accuracy of Aircraft Positioning by Navigational Aids Using Kalman Filter

*Signal Processing Symposium
September, 2019, Krakow, Poland*

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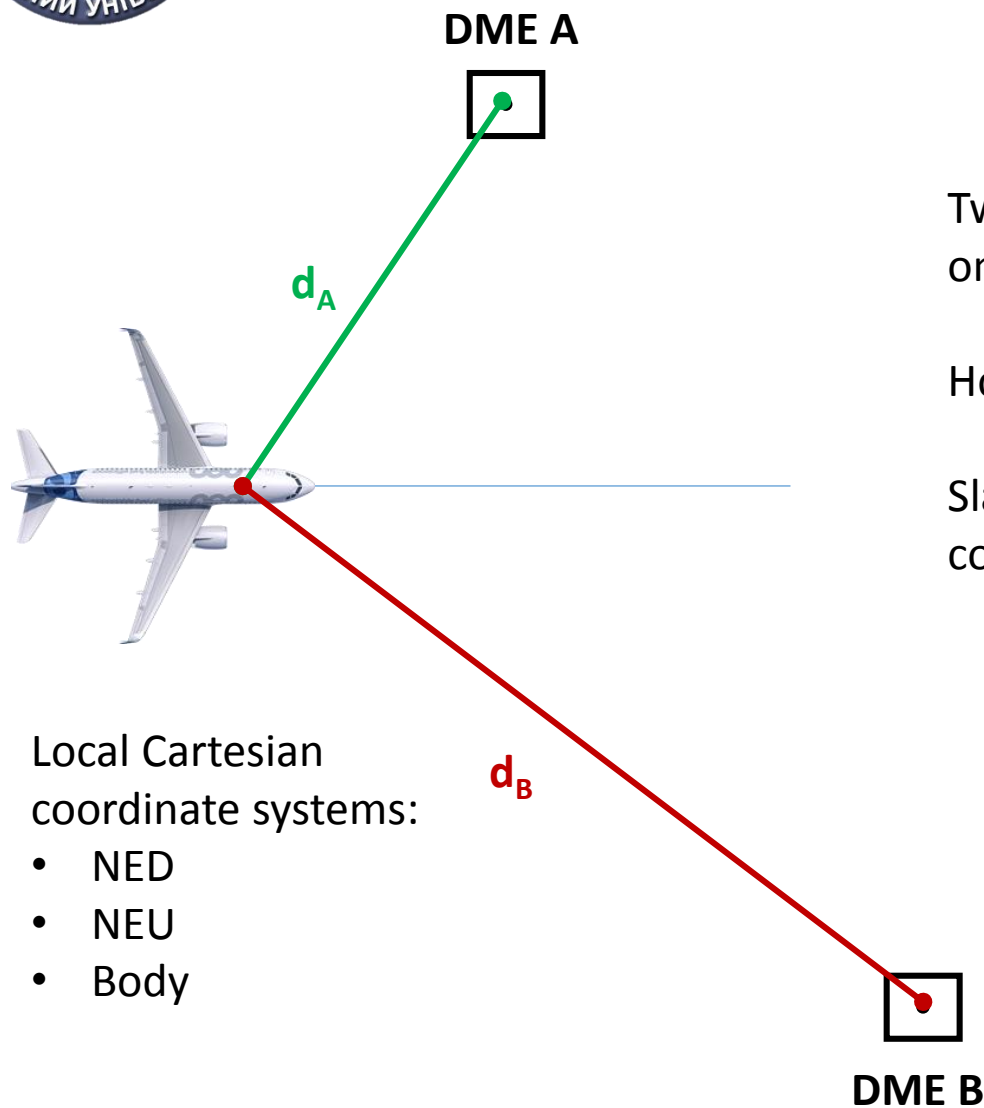


DME/DME





DME/DME (TOA) positioning method



Two ranges from Two DMEs support only Lateral Navigation (LNAV)

Horizontal distances are used for LNAV

Slant ranges from DME (d) should be converted to horizontal one (d_h)

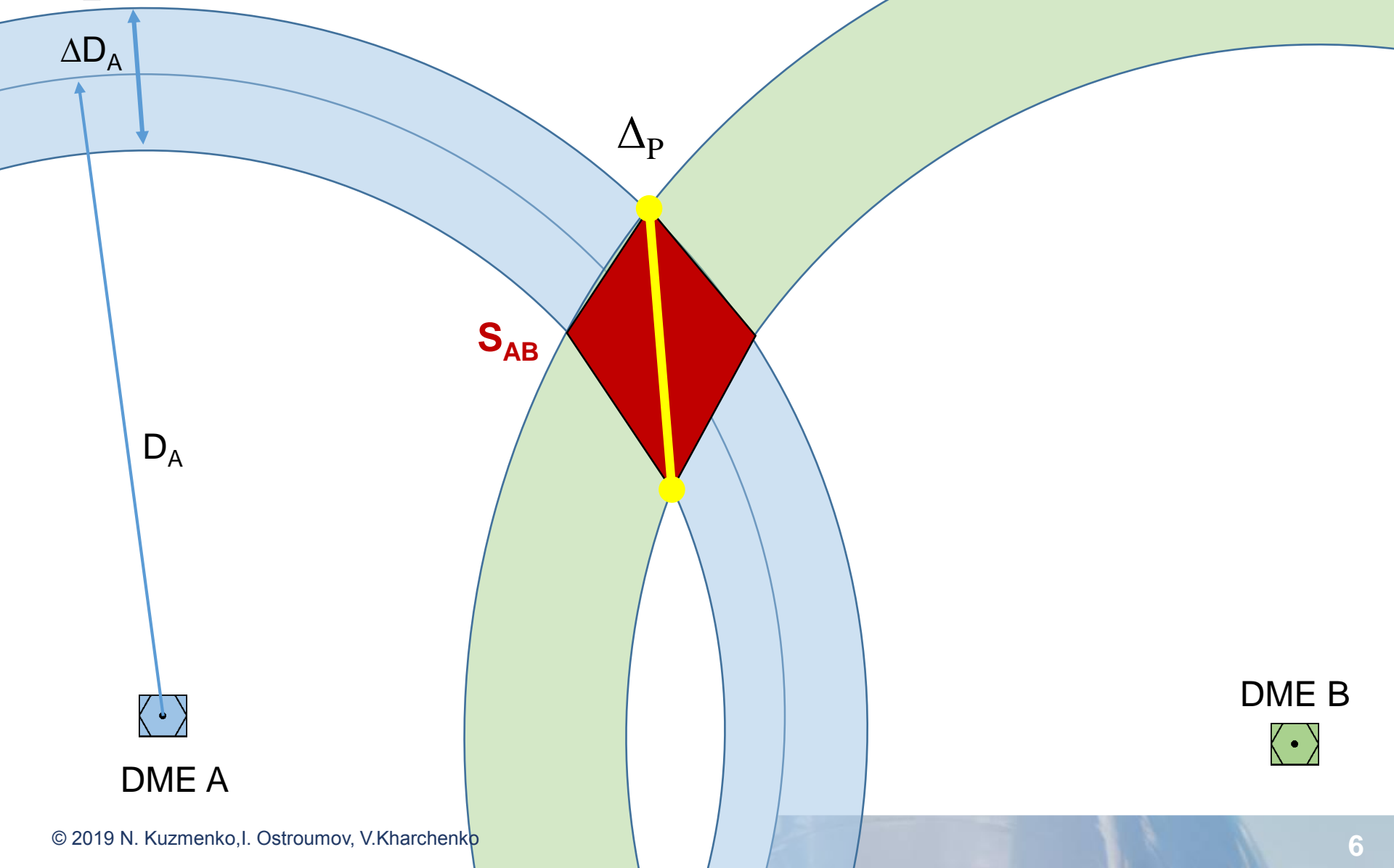
Local Cartesian coordinate systems:

- NED
- NEU
- Body

$$\begin{cases} d_{hA}^2 = (x_{DMEA} - x)^2 + (y_{DMEA} - y)^2 \\ d_{hB}^2 = (x_{DMEB} - x)^2 + (y_{DMEB} - y)^2 \end{cases}$$



Accuracy of DME/DME positioning





Error model

Results of measurements of distance D and angle α can be considered as an additive error:

$$D_i = D_i' + \xi_D,$$

$$\alpha_i = \alpha_i' + \xi_\alpha, \quad i=1, \dots, N$$

where D_i' and α_i' are true values; ξ_D and ξ_α are random variables from NPDF, N is a number of measurements.

In NPDF a mean-squared deviation defines an error model. An error of distance measurements:

$$\sigma_{DME}^2 = \sigma_{sis}^2 + \sigma_{air}^2$$

Airborne interrogator error

$$\sigma_{air} = \max\{0.085\text{NM}; 0.125\% R\},$$

where R is measured distance.

Signal in space propagation error

$$\sigma_{sis}^2 = \sigma_f^2 + \sigma_d^2 + \sigma_p^2$$

where

σ_f^2 is an error of fixation of the navigation signal arrival to the ground part of the equipment;

σ_d^2 is an error of measuring time delay of the navigation signal in the ground part of the DME equipment;

σ_p^2 is an error associated with the propagation of radio waves in space.



Error filtering

The idea of error filtering is grounded on applying an object model and statistical analysis of measurements in order to predict investigated parameter value at time of the next measurement. Next measured and predicted values are used in filtering.

We consider usage of α - β - γ filter in the form of linear **Kalman** filter to reduce noise in distance and angle measurements.

$$X_i = \Phi_{i-1} X_{i-1} + w_{i-1}$$

where X_i is a state matrix at time t_i ; Φ_{i-1} is a transformation matrix from previous state $i-1$ to current state i ; w_{i-1} is noise.

The state matrix $X_i = [x_i, v_i, a_i]^T$ includes smoothed parameter x_i , value of velocity v_i , and acceleration a_i . Also, we suppose that measurements are linearly connected with system state by the following model:

$$z_i = H_i X_i + u_i$$

where H is sensitivity matrix of measurement; u_i is an error of measurement.

We consider sensitivity matrix in the form: $H = [1, 0, 0]$, because we measure x_i value and estimate velocity and acceleration.



Error filtering

The extrapolated system state can be obtained as follows:

$$X_i^e = \Phi_{i-1} X_{i-1},$$

$$\Phi = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

where

$T = t_i - t_{i-1}$ is a time difference between measurements,

$X_{i-1} = [x_{i-1}, v_{i-1}, a_{i-1}]^T$ is a state matrix at time t_{i-1} .

A covariance error matrix of extrapolation is:

$$P_i^e = \Phi_{i-1} P_{i-1} \Phi_{i-1}^T + Q_{i-1},$$

$$P_{i-1} = \text{diag}([\sigma_x^2, \sigma_v^2, \sigma_a^2]),$$

where σ_x^2 , σ_v^2 and σ_a^2 are dispersion of x , v and a estimation; Q_{i-1} is an intensity matrix of noise.

A state estimate observation update can be obtained as follows:

$$X_i = X_i^e + K_i [z_i - H X_i^e],$$

$$K_i = \begin{bmatrix} \alpha & \frac{\beta}{T} & \frac{2\gamma}{T^2} \end{bmatrix}^T$$

where K is Kalman gain matrix.

An updated covariance error matrix is :

$$P_i = [I - K_i H] P_i^e,$$

where I is ones matrix.



Coefficients of α , β , γ

$$\alpha = 1 - r^2$$

$$\beta = 2(2 - \alpha) - \sqrt[4]{1 - \alpha},$$

$$\gamma = \frac{\beta^2}{2\alpha}$$

Damping coefficient:

$$r = \frac{(4 + \Lambda) - \sqrt{(8\Lambda + \Lambda^2)}}{4}$$

Tracking index

$$\Lambda = \frac{T^2 \sigma_g}{\sigma_x}$$

where σ_g is guidance error; σ_x is error of measurement.

T. Jeong, A. W. Njonjo, and B. F. Pan. "A study on the performance comparison of three optimal Alpha-Beta-Gamma filters and Alpha-Beta-Gamma-Eta filter for a high dynamic target," TransNav: International Journal on Marine Navigation and Safety of Sea Transportation, vol. 11, issue 1, March 2017, pp.55–61.

L. Ting-En, J. Su, K. Yu, and K. Hsia, "Design of Adaptive Alpha-Beta-Gamma Filters with Fuzzy Systems." in The 17th National Conference on Fuzzy Theory and Its Applications, pp. 910-915, 2009.



Filtering scenarios

A. Lower level of sensor data filtering scenario

At this level we use Kalman filter for sensor data filtering to reduce noise.

B. Upper level of trajectory data filtering scenario

In this scenario we use raw sensors data in positioning algorithm and apply filtering for track data.

Trajectory data

We use trajectory data of Embraer ERJ-145 aircraft for numerical demonstration. The data transmitted from on-board ATRBS of mode “ES” has been received and processed. Trajectory of “AUI79” flight from “KBP” to “IFO” on February 9, 2019 has been obtained

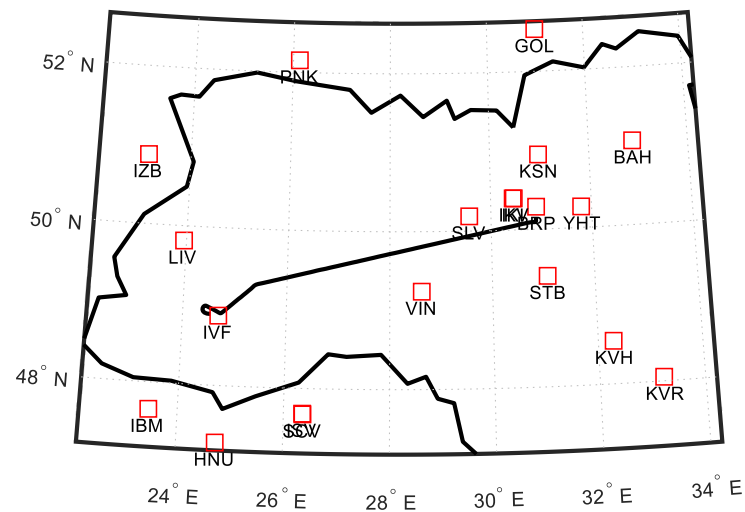


Fig. 1. Track of “AUI79” from “KBP” to “IFO” on February 9, 2019



Representation of DMEs data

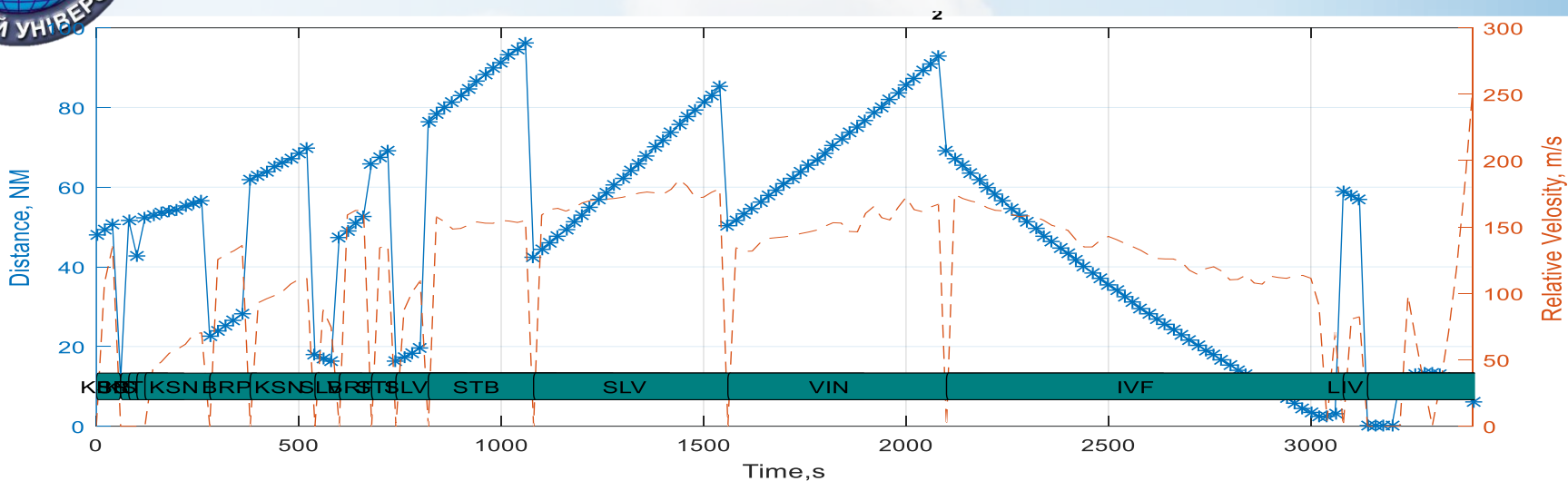


Fig. 1. Theoretical data of the first DME

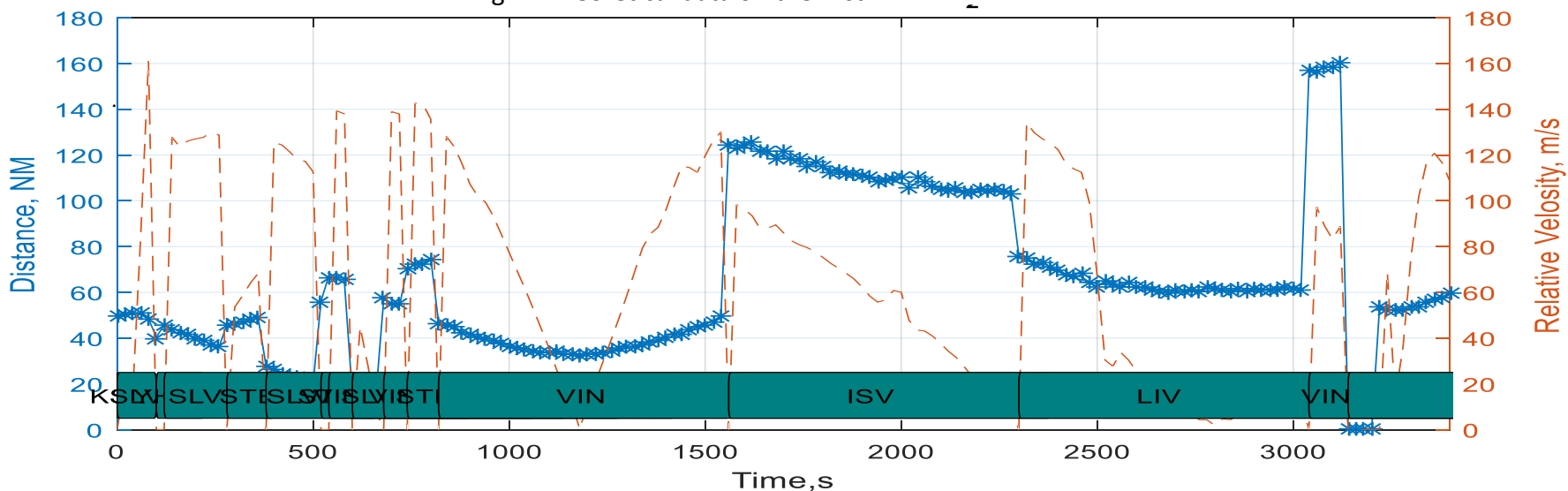


Fig. 1. Theoretical data of the second DME



A. Lower level of sensor data filtering scenario

The main problem in filtering measurements of navigational aids data is switching between different ground stations, because the value of distance changes significantly at the point of change. Therefore, filtering may be considered only within the time of one DME service to reduce interpolation error at the beginning of each ground station tracking.

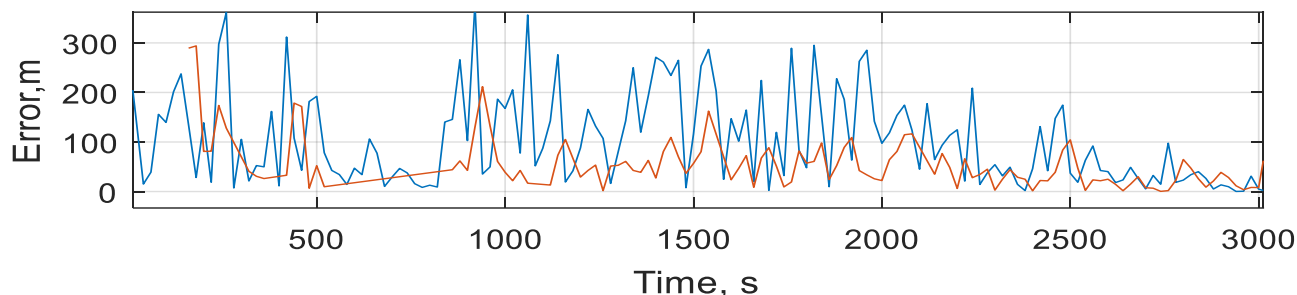


Fig. 1. Measured and filtered values of the first DME data

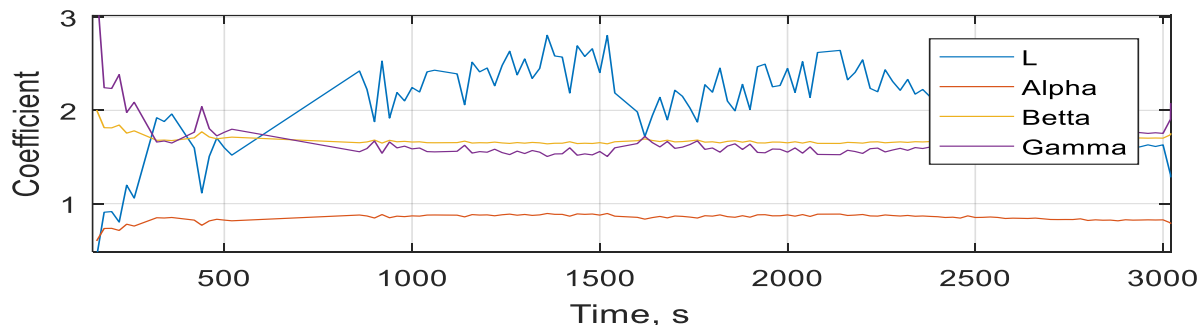


Fig. 2. α , β , γ coefficients

Results of error filtering is shown in Fig. 5 indicate reduction of standard deviation value from 112 m to 91 m that is approximately 19% of the efficiency.



B. Upper level of trajectory data filtering scenario

The result of positioning by DME/DME is a set of airplane coordinates in Cartesian coordinate system. We use local North-East-Down coordinate system (NED) for calculation with a reference point at “KSN” DME located at 50.9460 N latitude, 30.9778 E longitude, and 146.3 m altitude (WGS 84).

We use next navigation equation to find an aircraft location

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} = \begin{bmatrix} \frac{x_i - x_{DMEA}}{d_{DMEAi}} & \frac{y_i - y_{DMEA}}{d_{DMEAi}} \\ \frac{x_i - x_{DMEB}}{d_{DMEBi}} & \frac{y_i - y_{DMEB}}{d_{DMEBi}} \end{bmatrix}^{-1} \begin{bmatrix} d_{DMEAi} - d_{DMEA} \\ d_{DMEBi} - d_{DMEB} \end{bmatrix}$$

$$x_i = x_{i-1} + \Delta x_{i-1},$$

$$y_i = y_{i-1} + \Delta y_{i-1},$$

where x_{DMEA} , y_{DMEA} , x_{DMEB} , y_{DMEB} , are coordinates of DME A and DME B location in local NED coordinate system; d_{DMEA} , d_{DMEB} are measured distances from DME A and B correspondingly; x_i, y_i are solution of navigation equation at i^{th} iteration; d_{DMEAi} , d_{DMEBi} are calculated distances in horizontal plane between point of solution (x_i, y_i) and DMEs location; Δx_i and Δy_i are errors of position detection at i^{th} iteration.

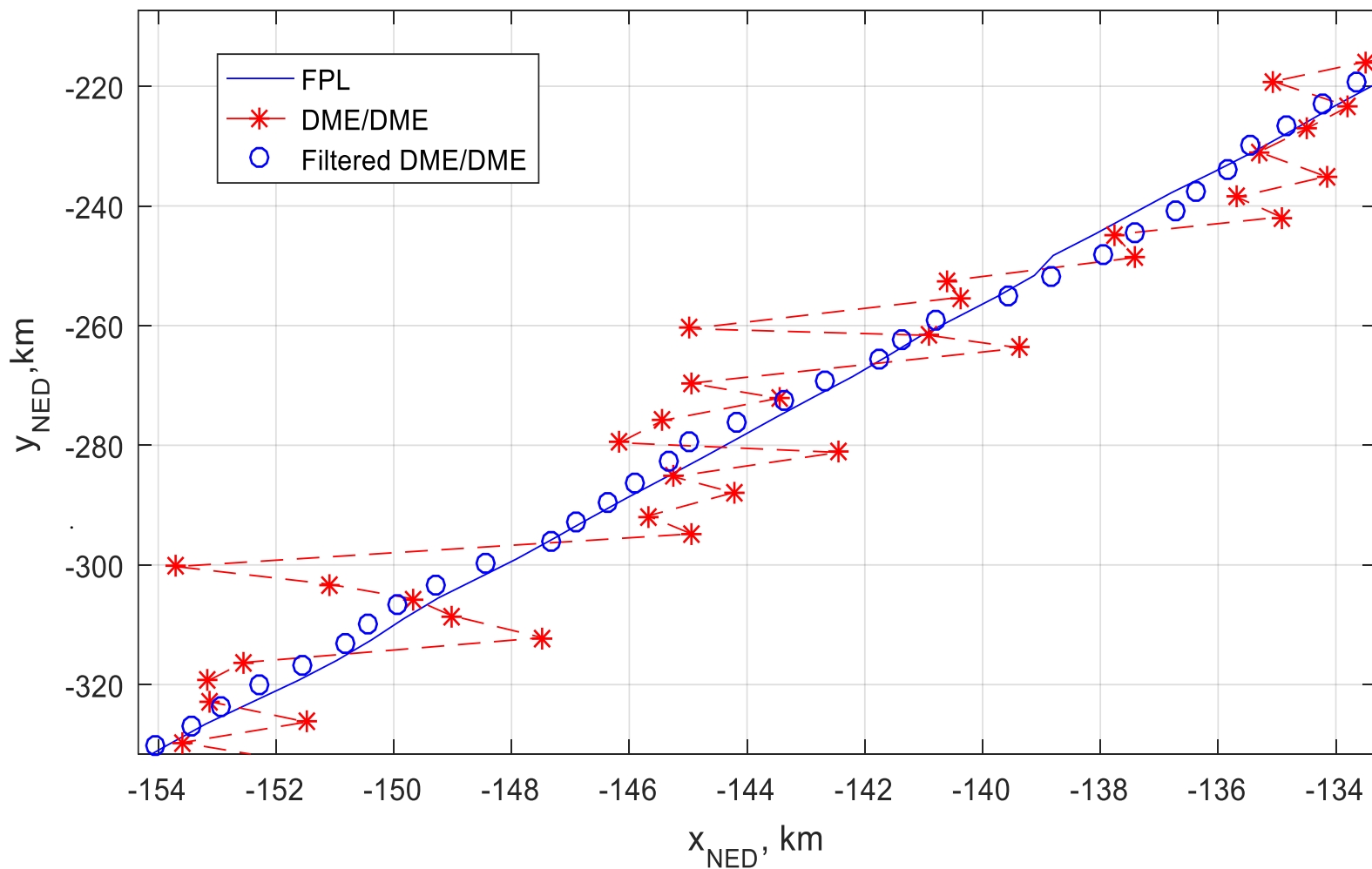
Finding a solution is an iterative process which continues until certain level of accuracy is satisfied:

$$\Delta x_{i-1}^2 + \Delta y_{i-1}^2 \leq \xi_{max}^2$$

where $\xi_{max} = 10^{-10}$ is an accuracy of navigation equation solution.



Upper level of trajectory data filtering





Results

Comparison of positioning accuracy for both levels by statistics indicates, that Standard deviation error is reduced at about 25% for upper level of trajectory data processing and about 14% for low level of filtering.

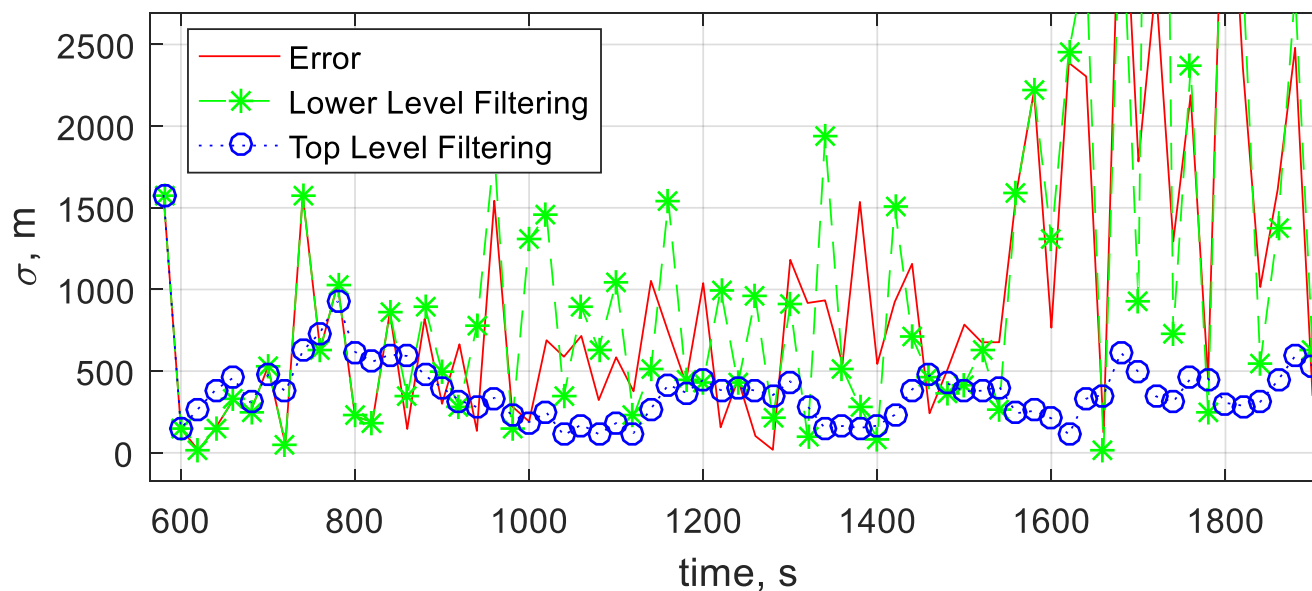


Fig. Performance of trajectory filtering



Conclusions

Errors of measurements in DME or VOR have significant impact on the total error of aircraft positioning by navigational aids. We analyzed implementation of α - β - γ filter in the form of linear Kalman filter in the task of positioning by navigational aids. Results of filtering at upper level of trajectory data processing show better performance in comparison with direct filtering at lower level of measurements for positioning by DME/DME. Low positioning accuracy is result of pair changes during en-route phase of flight. Also, filter requires at least three measurements for each DME to begin its operation. Therefore, noise reduction at the beginning of DME tracking is unavailable at lower level of filtering and number of DME pair changes is connected with performance of positioning. Obtained results indicate a possibility of Kalman filter implementation at the upper level of trajectory data processing for noise reduction and is important for the development of Alternative Positioning, Navigation, and Timing technologies for future air navigation system.