



Optimal Pair of Navigational Aids Selection

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Aircraft on-board equipment for positioning

Global Navigation Satellite System
(GPS, GLONASS, GALILEO, BeiDou)

Inertial Navigation System
(IRS, AHRS)

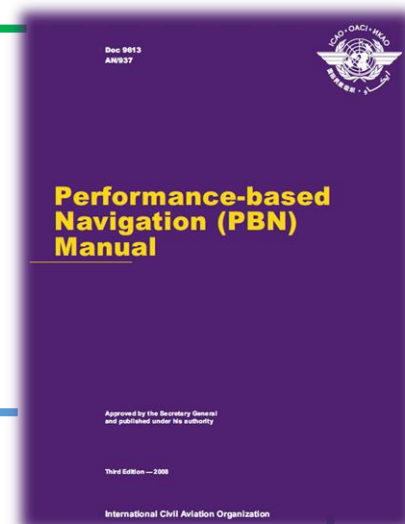
Position detection by pair of DME (DME/DME)

Position detection by pair of VOR and DME (VOR/DME)

Positioning by Navigational Aids in Flight Management System

Compatible
with PBN
(RNAV 1)

ICAO
DOC 9613

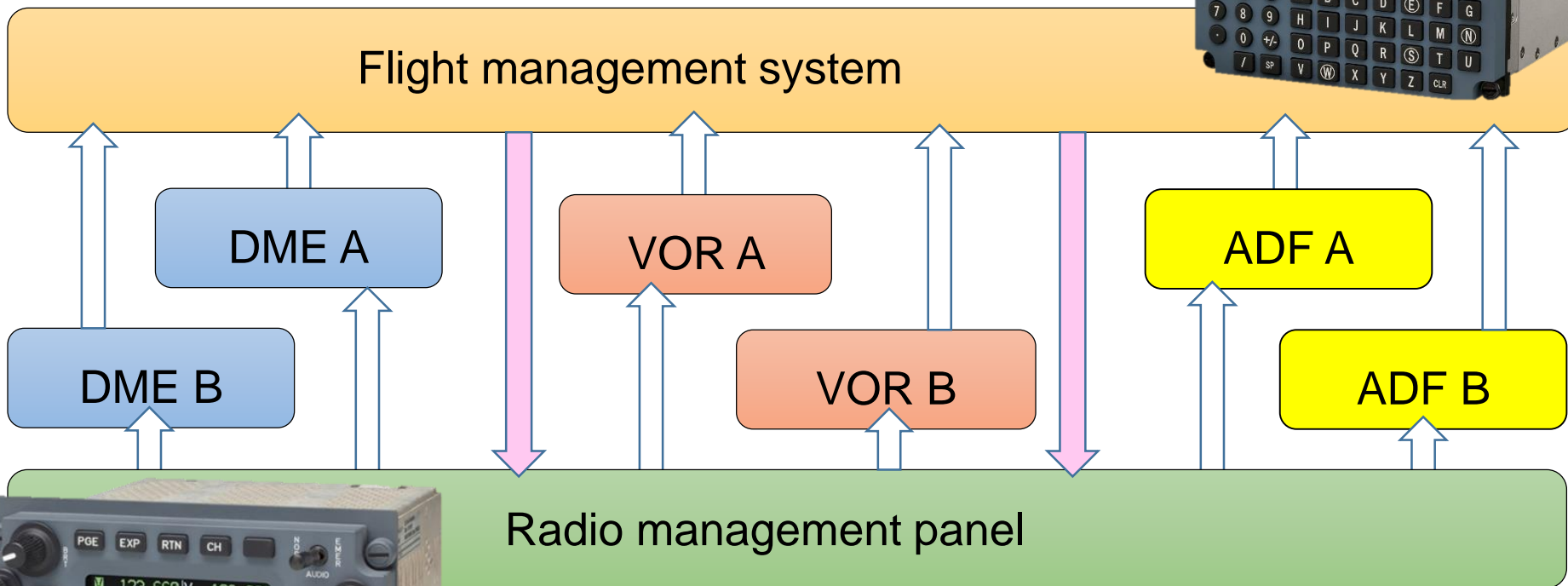


Errors of Global Navigation Satellite Systems

- large spatial variation in ionospheric delay
- **Interference from ground based radio equipment**
- **jamming of radio waves**
- Refraction from obstacles (multipath error)
- Geometric factor of the satellite segment



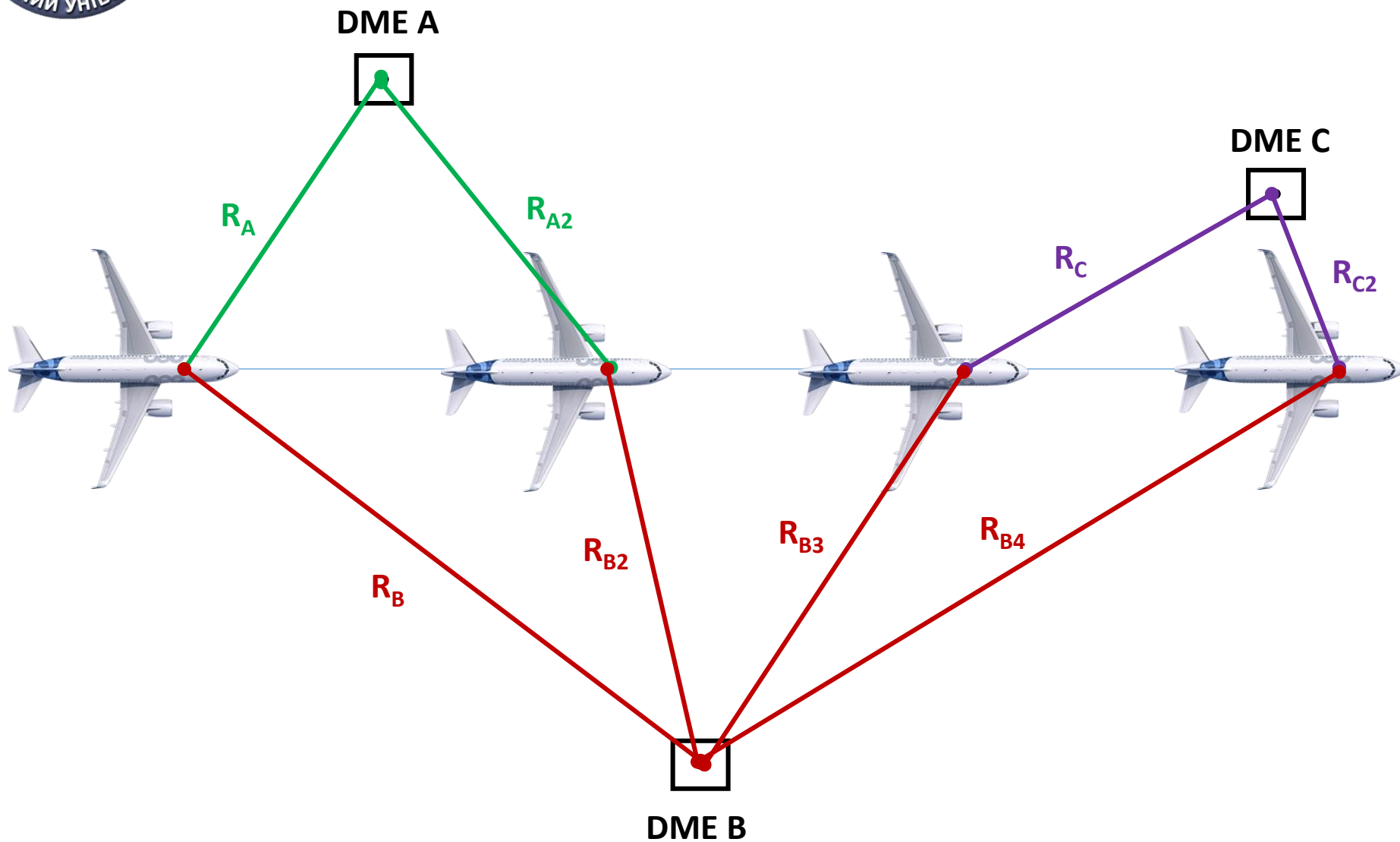
Interchange of NAVAIDS data of on-board equipment



Only pair of DME/DME equipment is in use for positioning function.

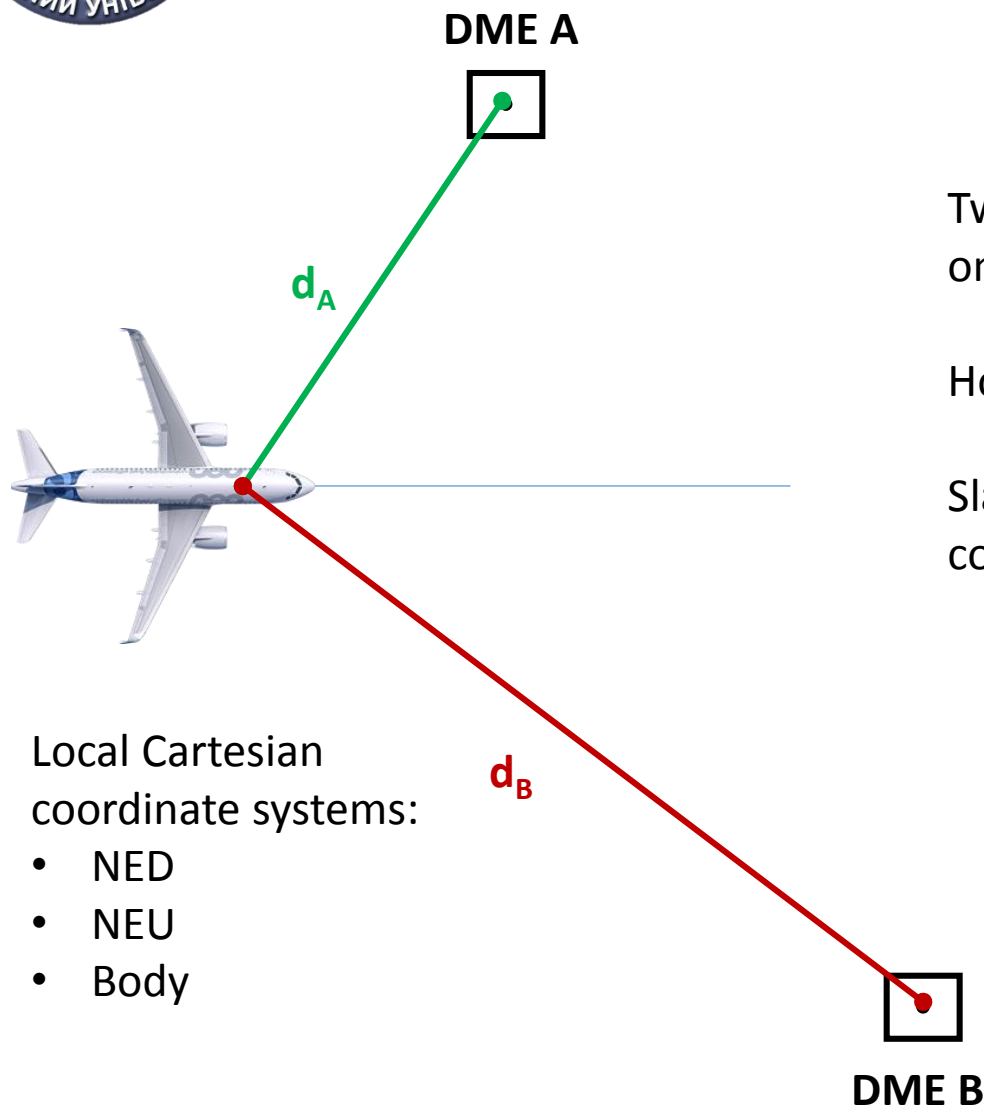


DME/DME





DME/DME (TOA) positioning method



Two ranges from Two DMEs support only Lateral Navigation (LNAV)

Horizontal distances are used for LNAV

Slant ranges from DME (d) should be converted to horizontal one (d_h)

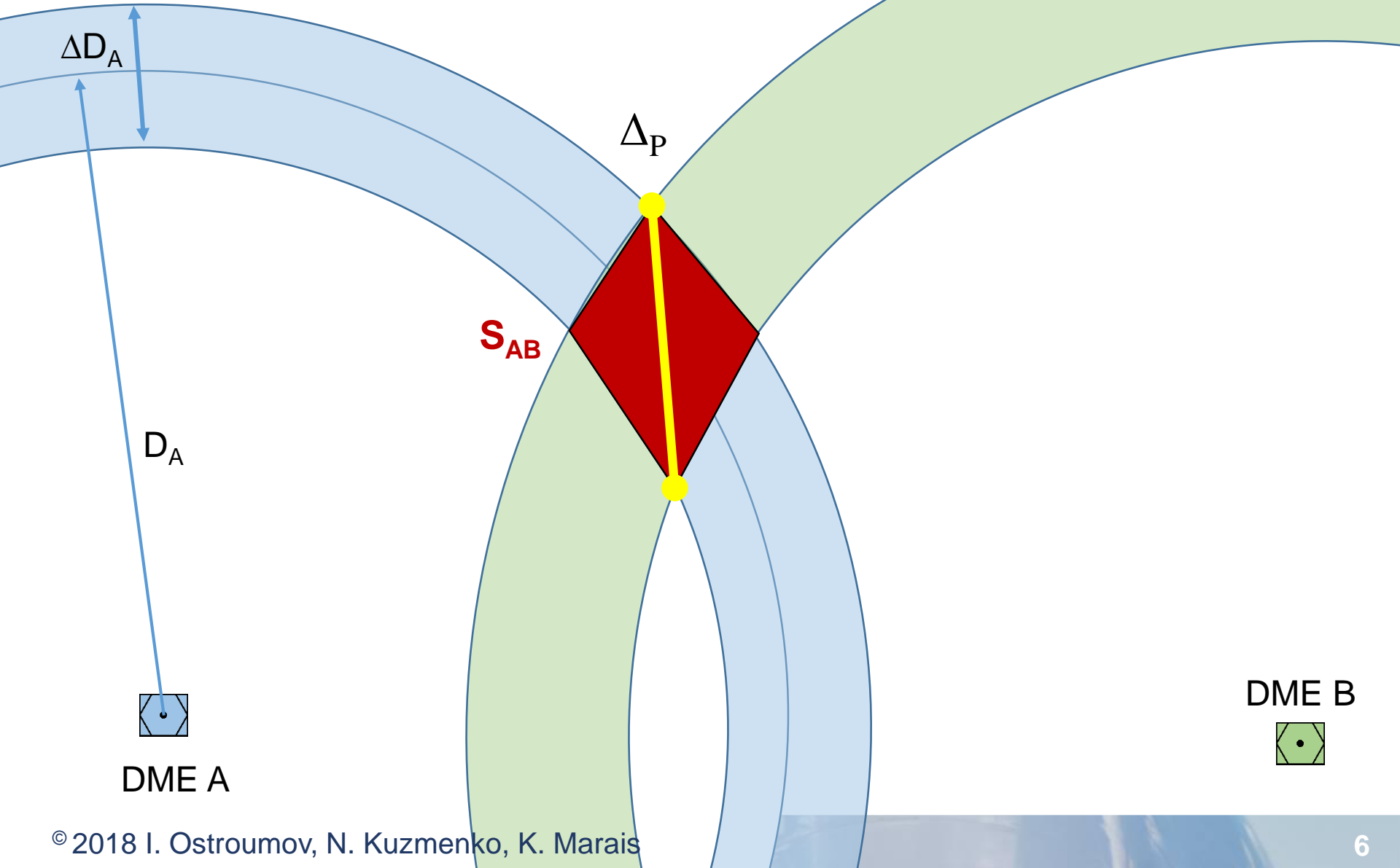
Local Cartesian coordinate systems:

- NED
- NEU
- Body

$$\begin{cases} d_{hA}^2 = (x_{DMEA} - x)^2 + (y_{DMEA} - y)^2 \\ d_{hB}^2 = (x_{DMEB} - x)^2 + (y_{DMEB} - y)^2 \end{cases}$$



Accuracy of DME/DME positioning





Accuracy of DME

$$\sigma_P = \frac{\sqrt{\sigma_{DMEA}^2 + \sigma_{DMEB}^2}}{\sin(\alpha_{AB})}$$

$$\sigma_{DMEA,B}^2 = \sigma_{sis}^2 + \sigma_{air}^2$$

σ_{sis}^2 – Signal in space error

σ_{air}^2 – Error of airborne interrogator

$$\sigma_{sis} = 0.05 \text{ NM}$$

RTCA DO-189:

$$\sigma_{air} = 0.085 \text{ NM}$$

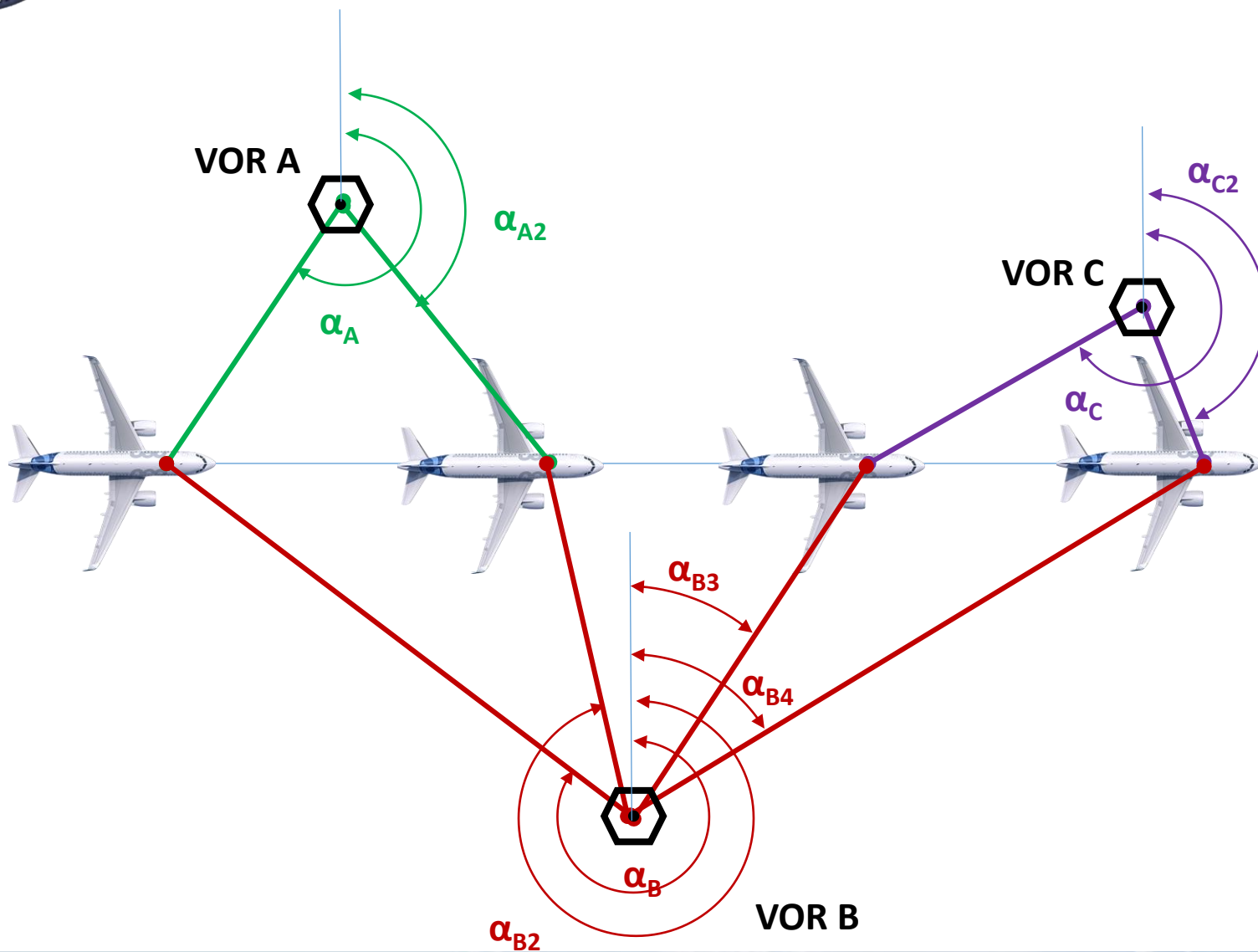
AC90-100A and ICAO DOC-9613:

$$\sigma_{air} = \max\{0.085 \text{ NM}; 0,125\% R\},$$

$$\sigma_P = \frac{\sqrt{2\sigma_{sis}^2 + \sigma_{airA}^2 + \sigma_{airB}^2}}{\sin(\alpha_{AB})}$$

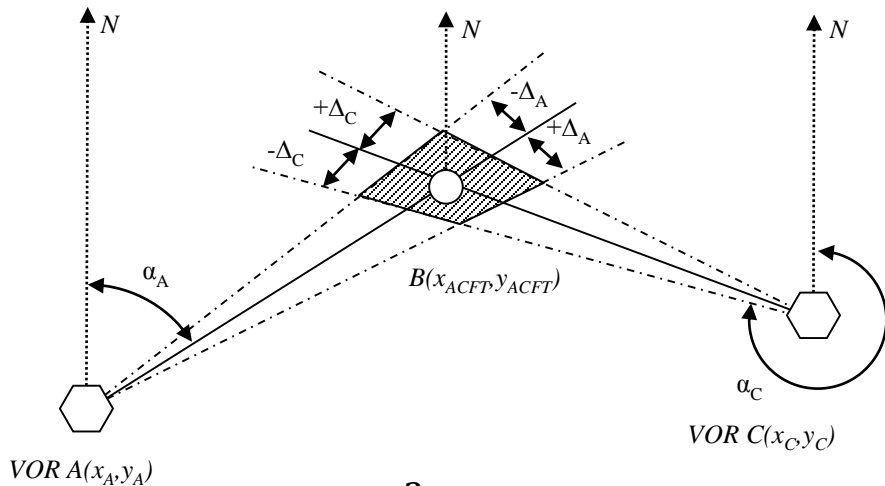


VOR/VOR





Error of positioning in VOR/VOR

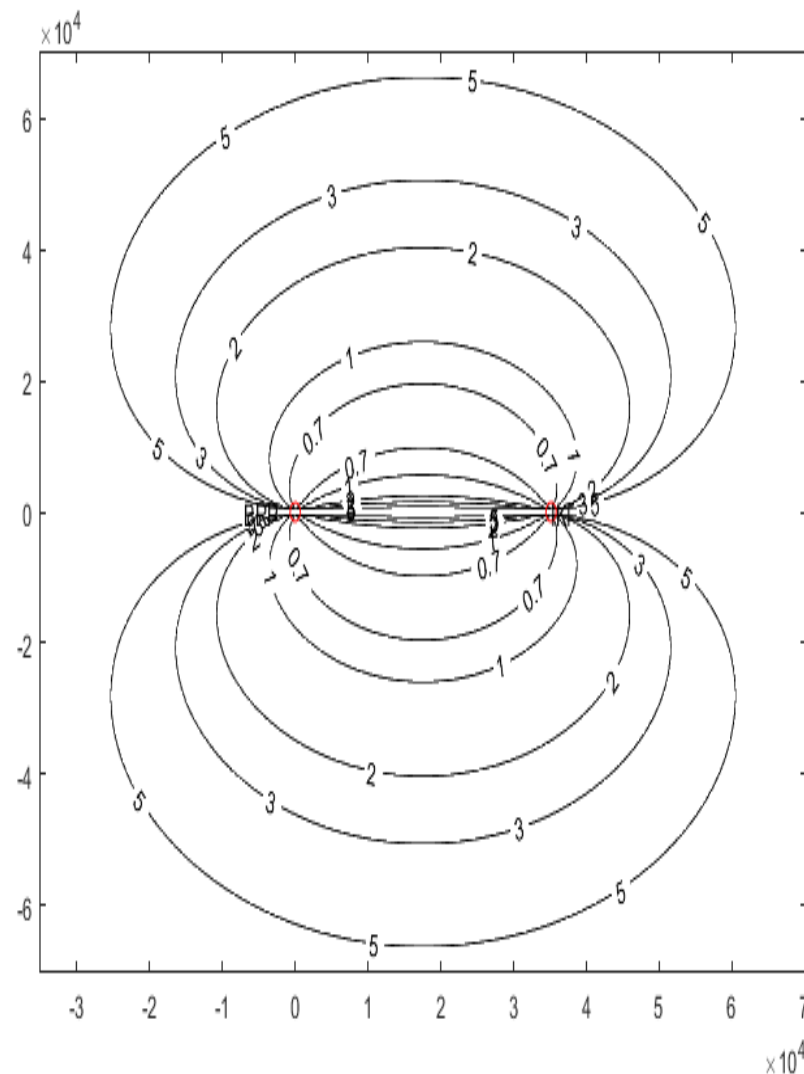
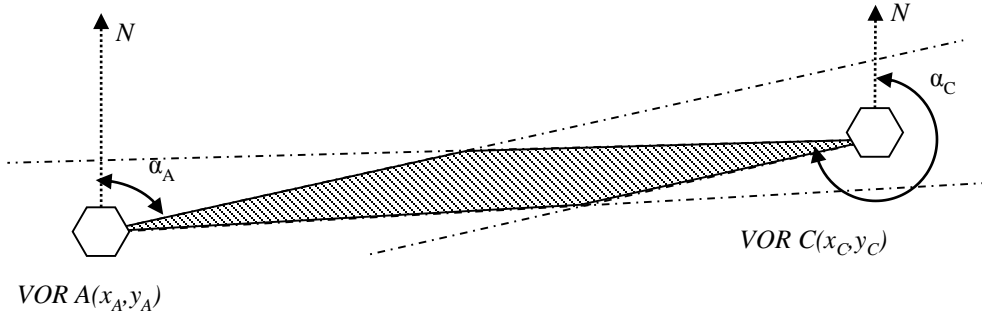


$$\sigma_p^2 = \frac{\sigma_\alpha^2}{\sin^2(\alpha)} (d_A^2 + d_B^2)$$

Where:

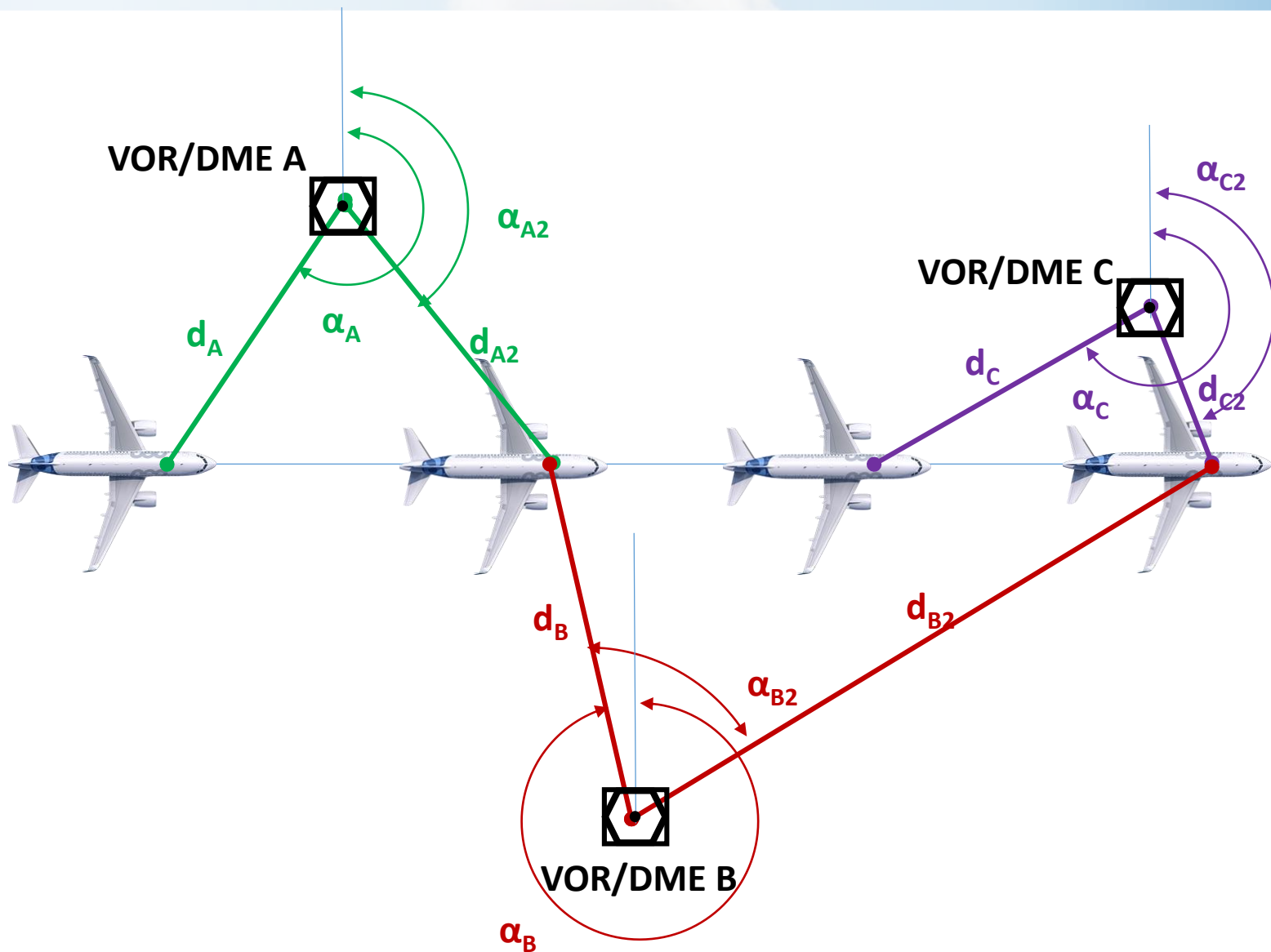
$$d_A = \frac{d \sin(\alpha_B)}{\sin(\alpha_A + \alpha_B)}$$

$$d_B = \frac{d \sin(\alpha_A)}{\sin(\alpha_A + \alpha_B)}$$



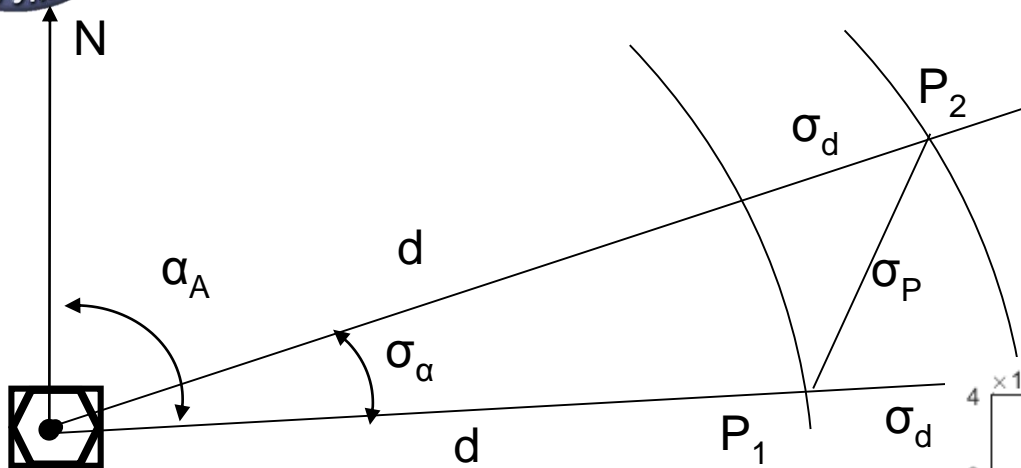


VOR/DME



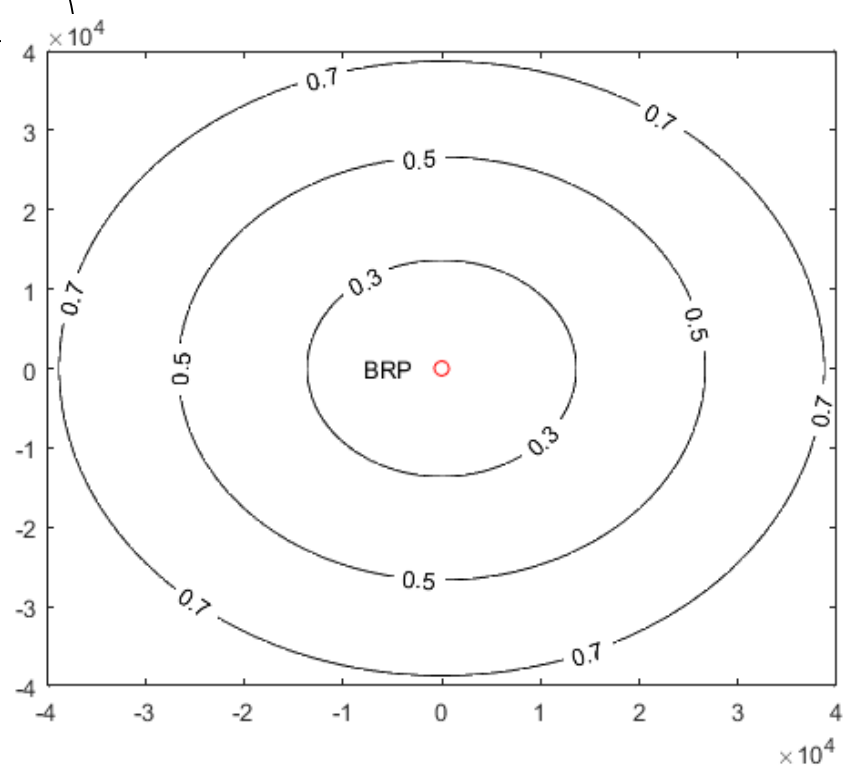


Error of positioning in VOR/DME



VOR/DME

$$\sigma_p^2 = \sigma_{DME}^2 + D^2 \sigma_{VOR}^2$$





Standard Service Volumes

Ground stations classification

T (Terminal)

From 1000 feet (305 m) AGL up to and including 12,000 feet (3,658 m) AGL at radial distances out to 25 nm (46 km).

L (Low Altitude)

From 1000 feet (305 m) AGL up to and including 18,000 feet (5,486 m) AGL at radial distances out to 40 nm (74 km).

H (High Altitude)

- From 1000 feet (305 m) AGL up to and including 14,500 feet (4,420 m) AGL at radial distances out to 40 nm (74 km).
- From 14,500 feet (4,420 m) AGL up to and including 60,000 feet (18,288 m) at radial distances out to 100 nm (185 km).
- From 18,000 feet (5,486 m) AGL up to and including 45,000 feet (13,716 m) at radial distances out to 130 nm (241 km).

AGL – Above Ground Level

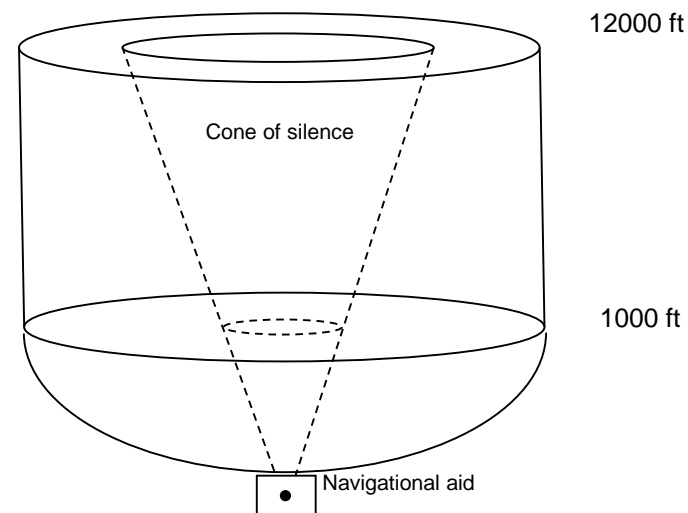
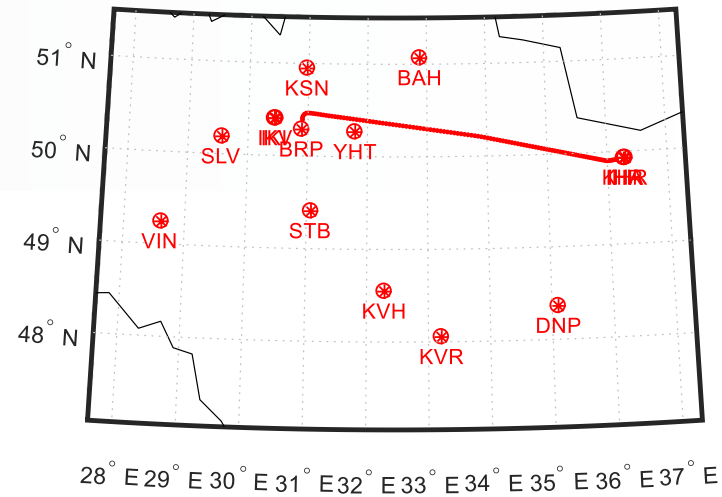


Fig. 12.1. Standard service volume of Terminal navigational aid



An Example

“AUI 25” Kyiv (UKBB)/Kharkiv (UKHH), May 14, 2018



28° E 29° E 30° E 31° E 32° E 33° E 34° E 35° E 36° E 37° E

Fig. 14.2. Representation of "AUI 25" flight track

n NAVAIDs generates m combinations of pairs:

$$m = n! / ((n-2)! \cdot 2) - 1$$

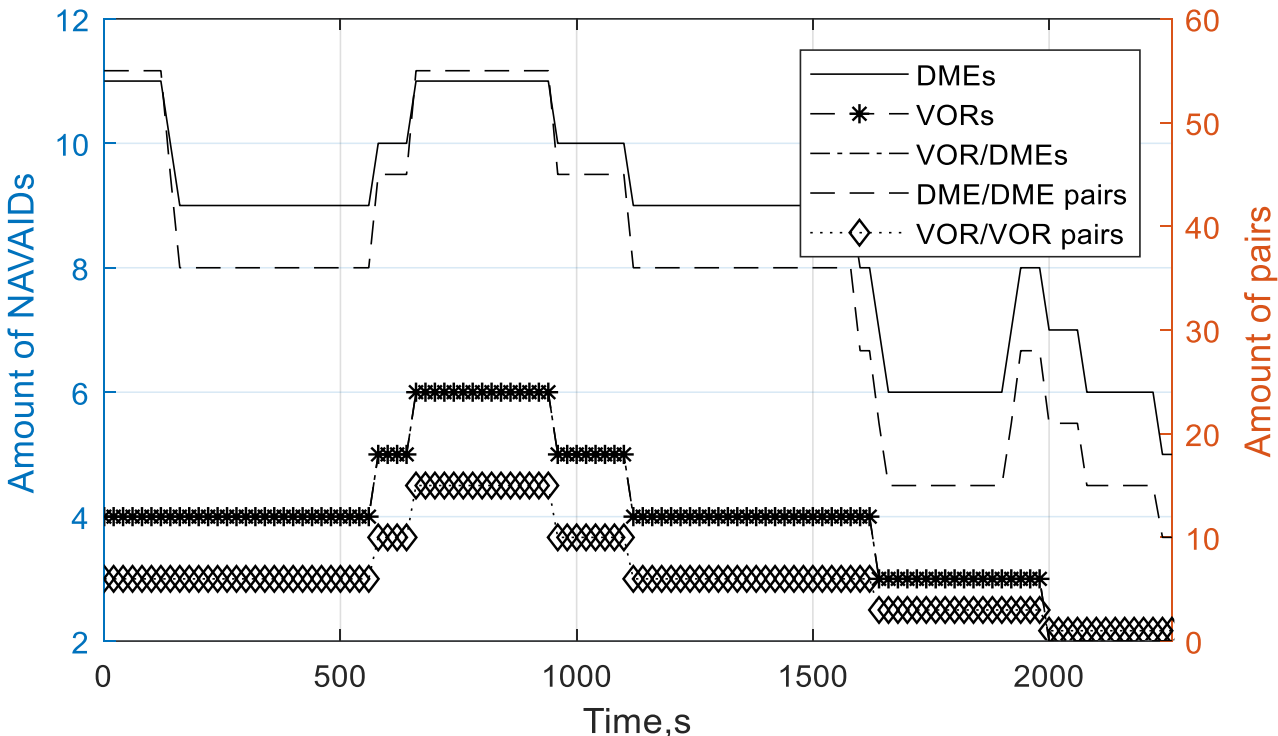


Fig. 14.1. The total amount of available DMEs, VORs and their pair combinations



Optimization problem

NAVAIDs optimal pair selection is a typical optimization problem which can be represented as an objective function and constraints in terms of integer linear programming

Objective function: $WX^T \rightarrow \min$,

Constraints: $AX^T \geq Nmax$

- by NAVAIDs availability in space

$GX^T \geq 30^\circ$

- by inclusion angle

- $GX^T \geq -150^\circ$

$IX^T=1$

- by number of solution

$X \geq 0$

$X \in \{0,1\}$

where

$X=[x_1, x_2, x_3, \dots, x_n]$ – binary matrix which includes indices of pairs;

$W=[W_{DME/DME}, W_{VOR/VOR}, W_{VOR/DME}]$ – weight matrix of each pair of NAVAIDs;

– for DME/DME: $W_{DME/DME}=[\sigma_{DME/DME 1}, \sigma_{DME/DME 2}, \sigma_{DME/DME 3}, \dots, \sigma_{DME/DME n}]$;

– for VOR/VOR: $W_{VOR/VOR}=[\sigma_{VOR/VOR 1}, \sigma_{VOR/VOR 2}, \sigma_{VOR/VOR 3}, \dots, \sigma_{VOR/VOR n}]$;

– for VOR/DME: $W_{VOR/DME}=[\sigma_{VOR/DME 1}, \sigma_{VOR/DME 2}, \sigma_{VOR/DME 3}, \dots, \sigma_{VOR/DME n}]$;

$A=[a_1, a_2, a_3, \dots, a_n]$ – availability binary matrix;

$G=[g_1, g_2, g_3, \dots, g_n]$ – matrix of inclusion angles.



Verifications

“AUI 25” Kyiv (UKBB)/Kharkiv (UKHH), May 14, 2018

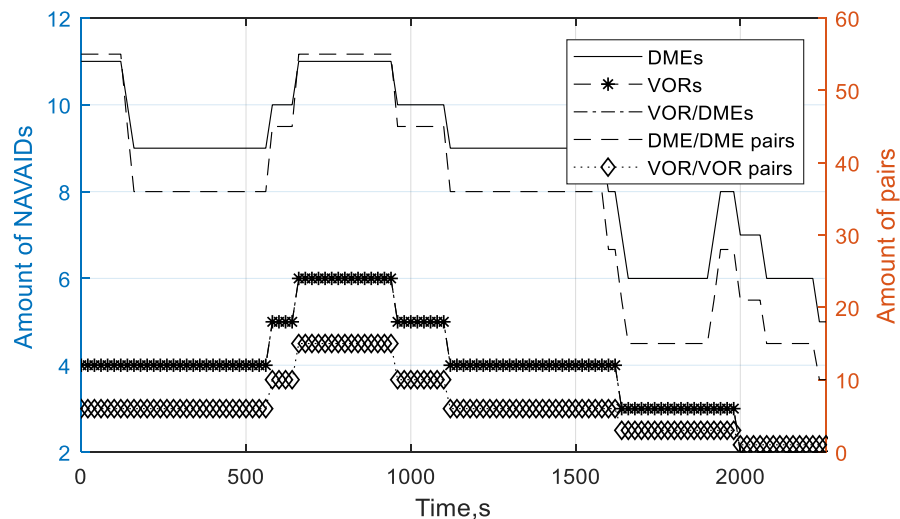


Fig. 16.1. The total amount of available DMEs, VORs and their pair combinations

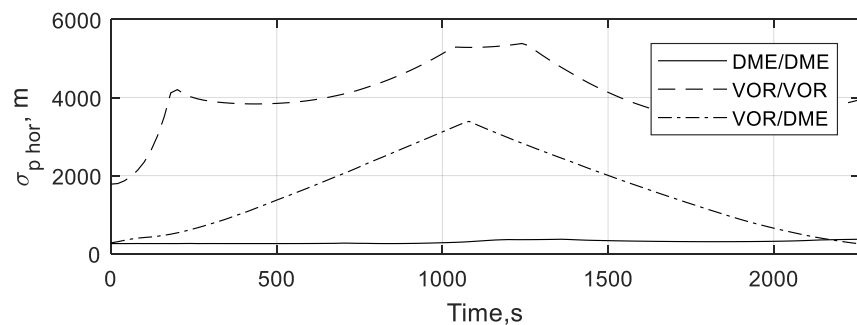


Fig. 16.2. Accuracy of different positioning methods

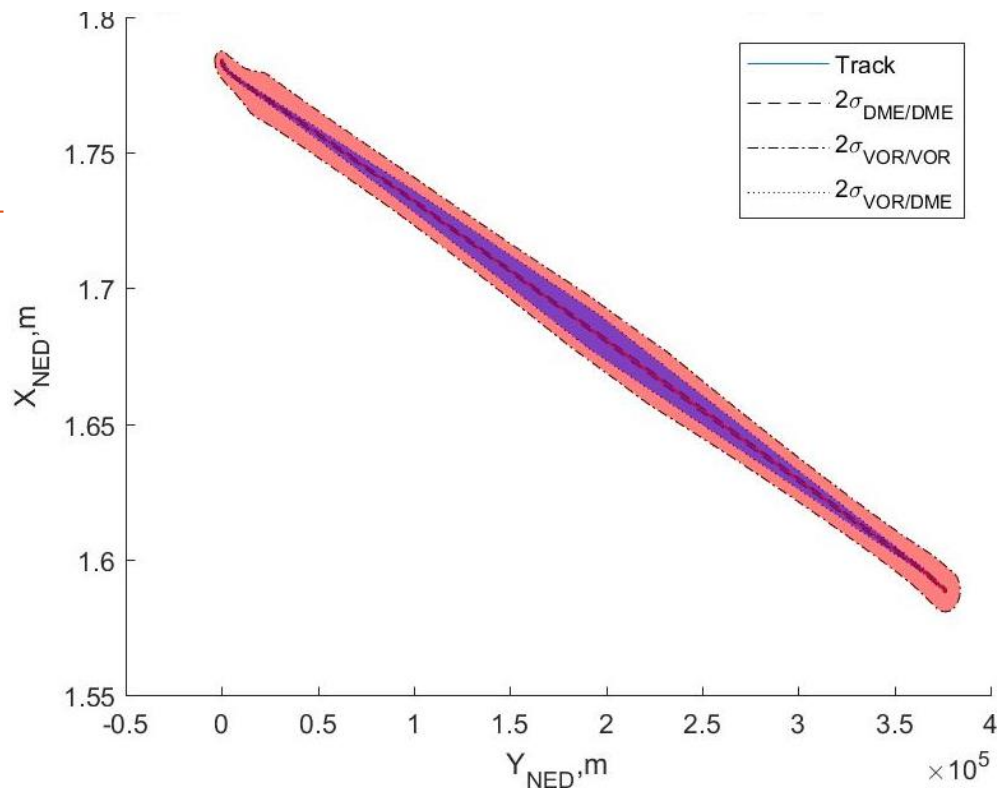


Fig. 16.3. Confidence band



DME/DME

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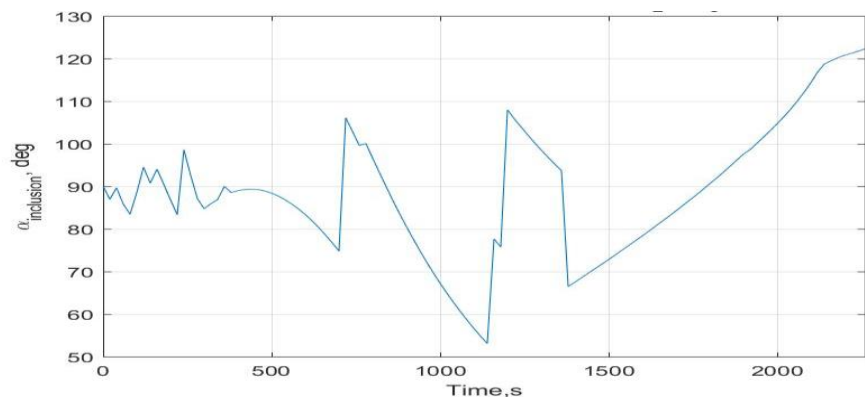


Fig. 17.1. Relative angle for optimal pair

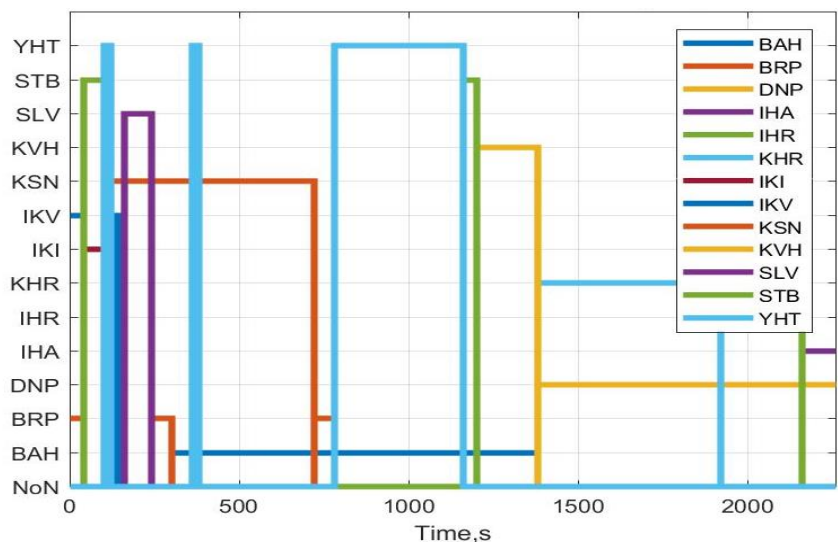


Fig. 17.2. Operational cycle

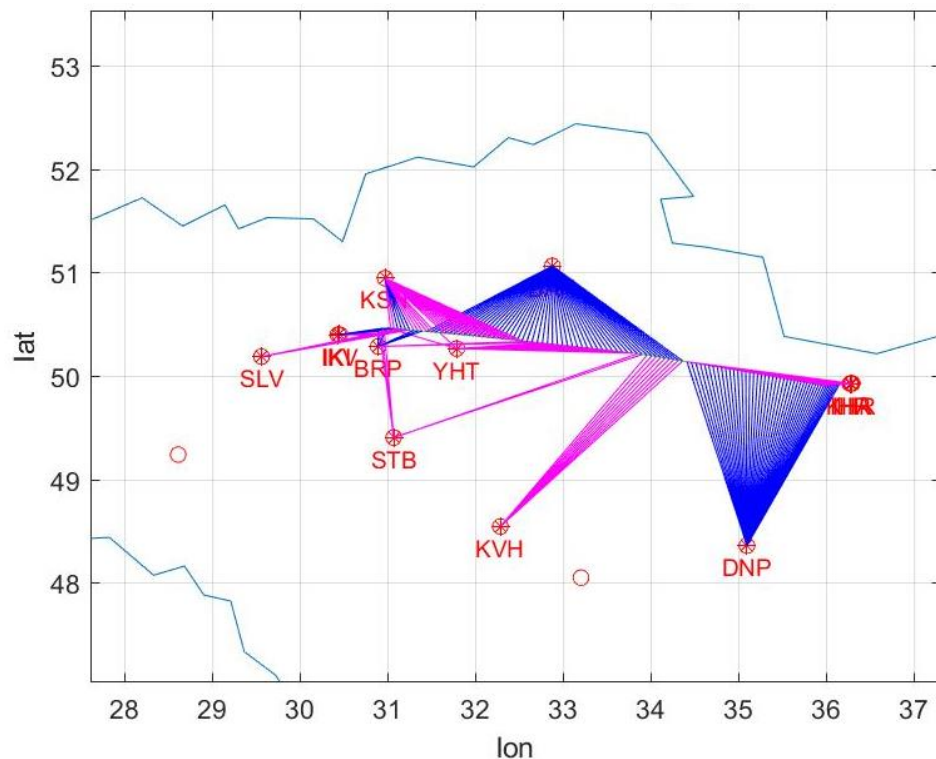


Fig. 17.3. Optimal pair of DME/DME



VOR/VOR

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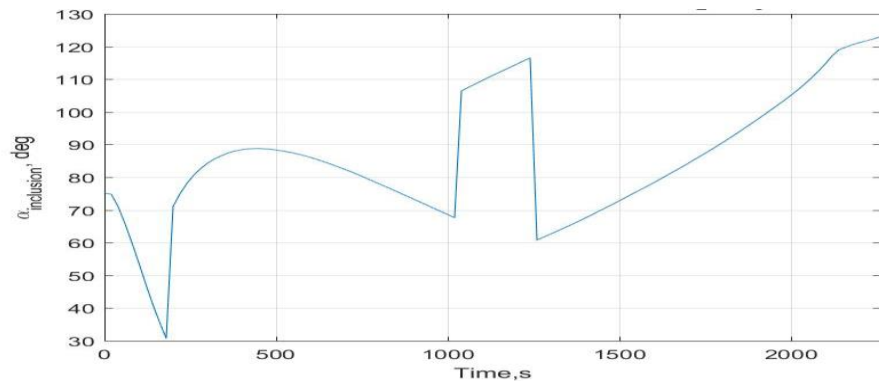


Fig. 18.1. Relative angle for optimal pair

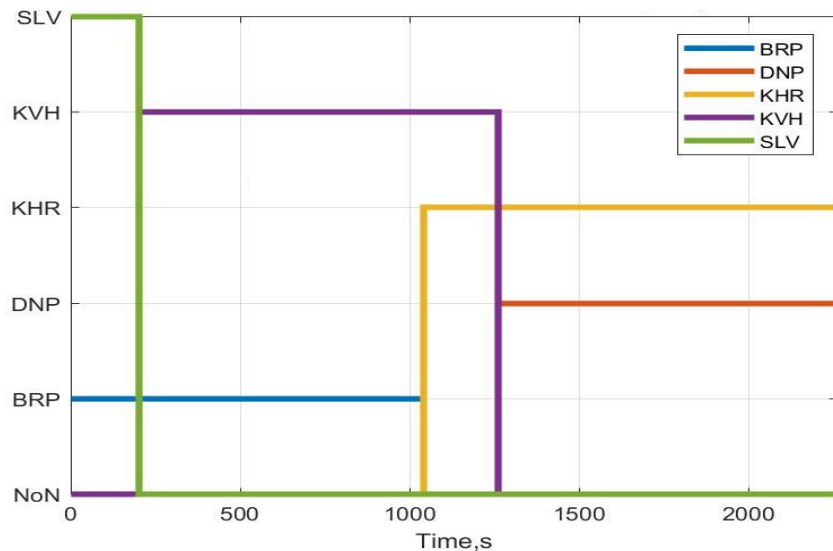


Fig. 18.2. Operational cycle

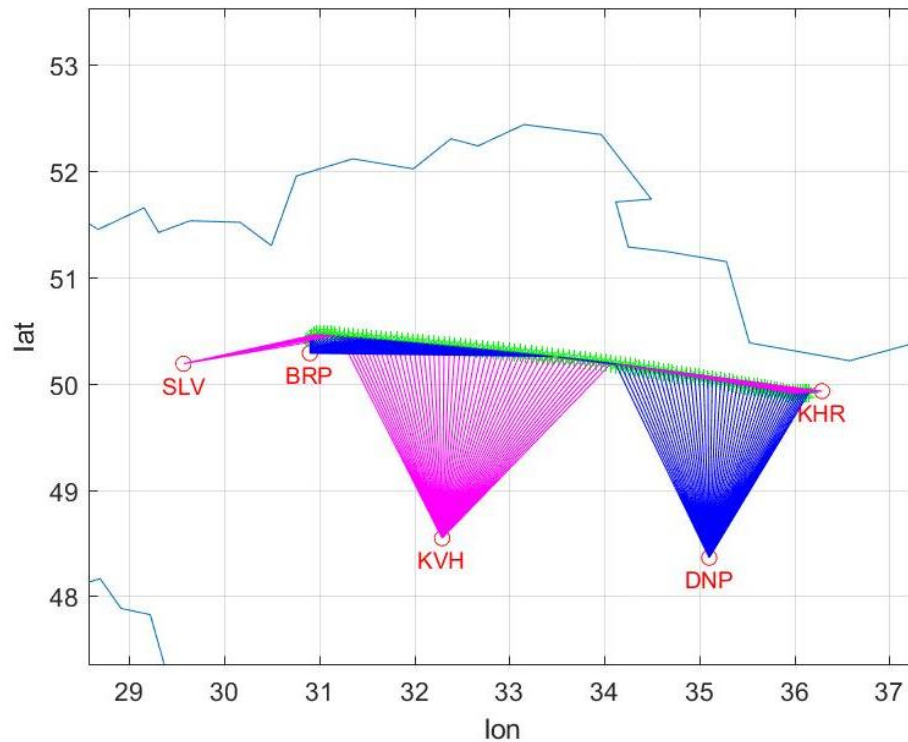


Fig. 18.3. Optimal pair



VOR/DME

“AUI 25” Kyiv (UKBB)/Kharkiv (UKHH), May 14, 2018

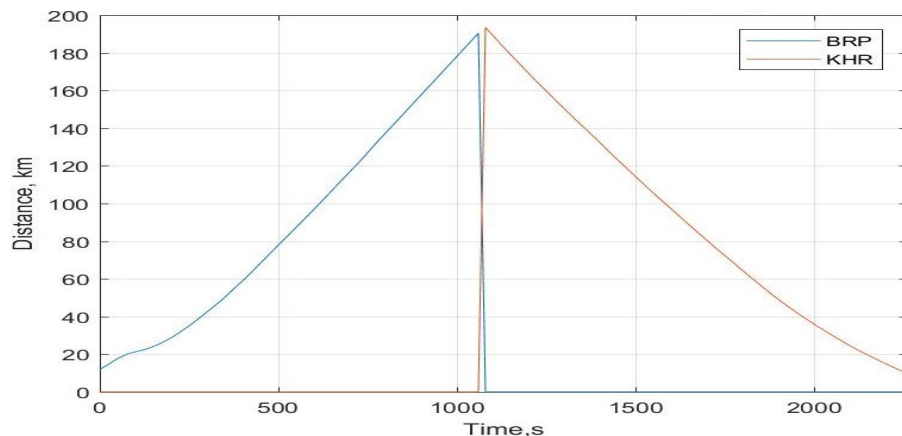


Fig. 19.1. Distance to optimal VOR/DME

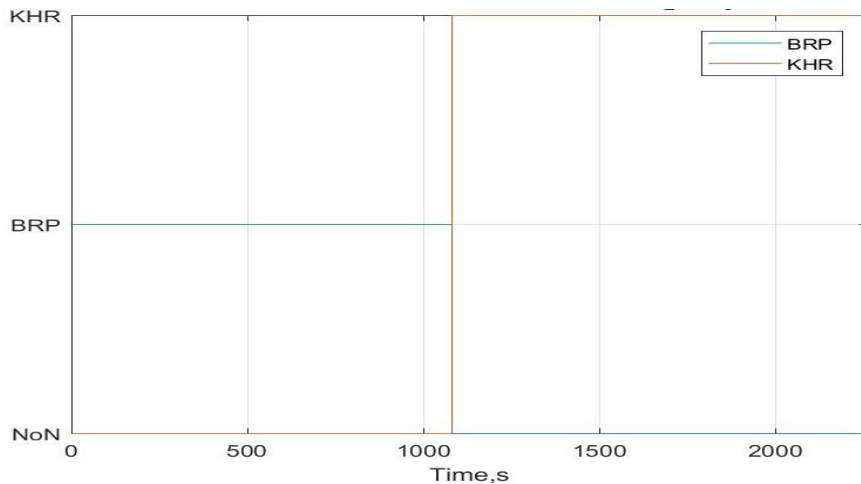


Fig. 19.2. Operational cycle

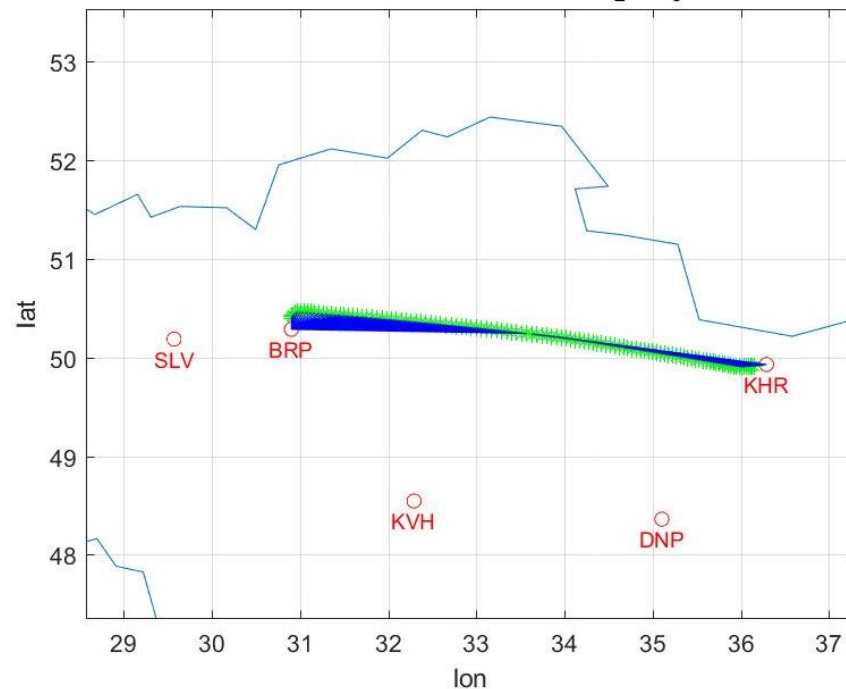


Fig. 19.3. Optimal pair of VOR/DME



The graph representation of switching pairs process

“AUI 25” Kyiv (UKBB)/Kharkiv (UKHH), May 14, 2018

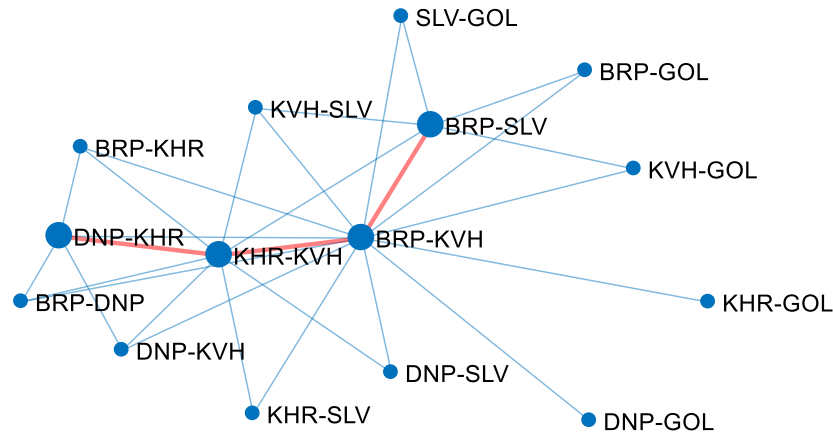


Fig. 20.1. The graph of VOR/VOR optimal pair switching

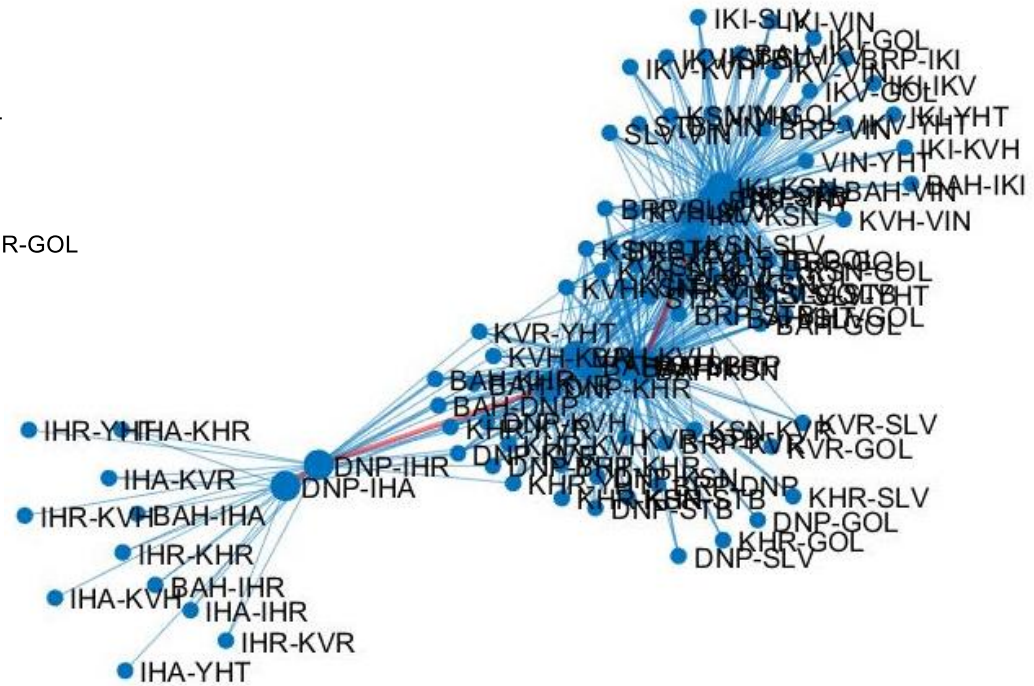


Fig. 20.3. The graph of DME/DME switching

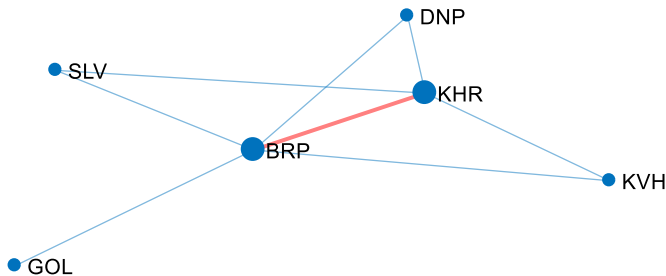


Fig. 20.2. The graph of VOR/DME switching



Conclusions

NAVAIDs optimal pair selection is a very important problem for aircraft positioning by DME and VOR data. Proposed approach considers optimization problem in terms of integer linear programming that combines all possible DME/DME, VOR/DME, and VOR/VOR available pairs to provide optimal positioning method and optimal pair information for its solution.

Results of computer-based simulation using “AUI 25” flight track data indicate an importance of optimal pair selection problem and proves efficiency of proposed approach. An accuracy of positioning by DME/DME method has the lowest value during almost the whole flight and guarantees flight performance within RNAV1 requirements. VOR/DME navigation indicates continuous increasing of positioning error moving from ground station. Results of simulation show that VOR/DMEs of involved airports were selected as optimal for chosen flight track. Positioning by VOR/VOR requires changes of selected pair three times and can be referred as the most inaccurate method.