



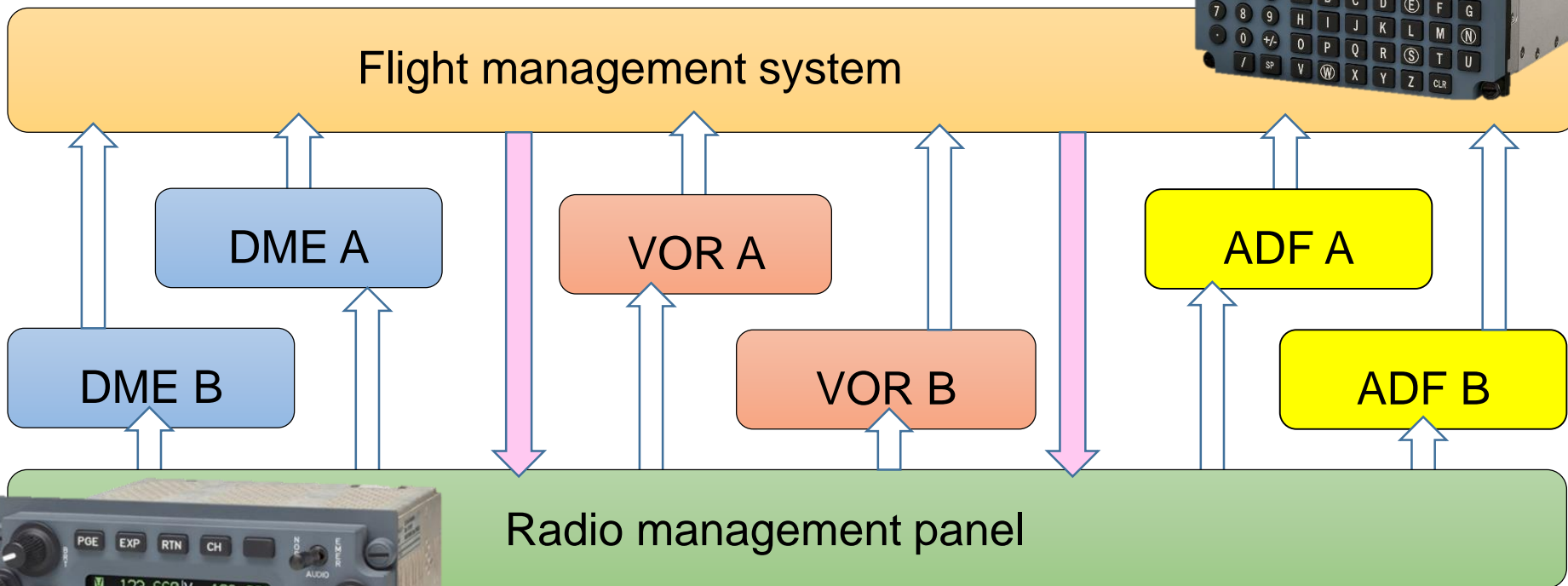
# An Accuracy and Availability Estimation of Aircraft Positioning by Navigational Aids

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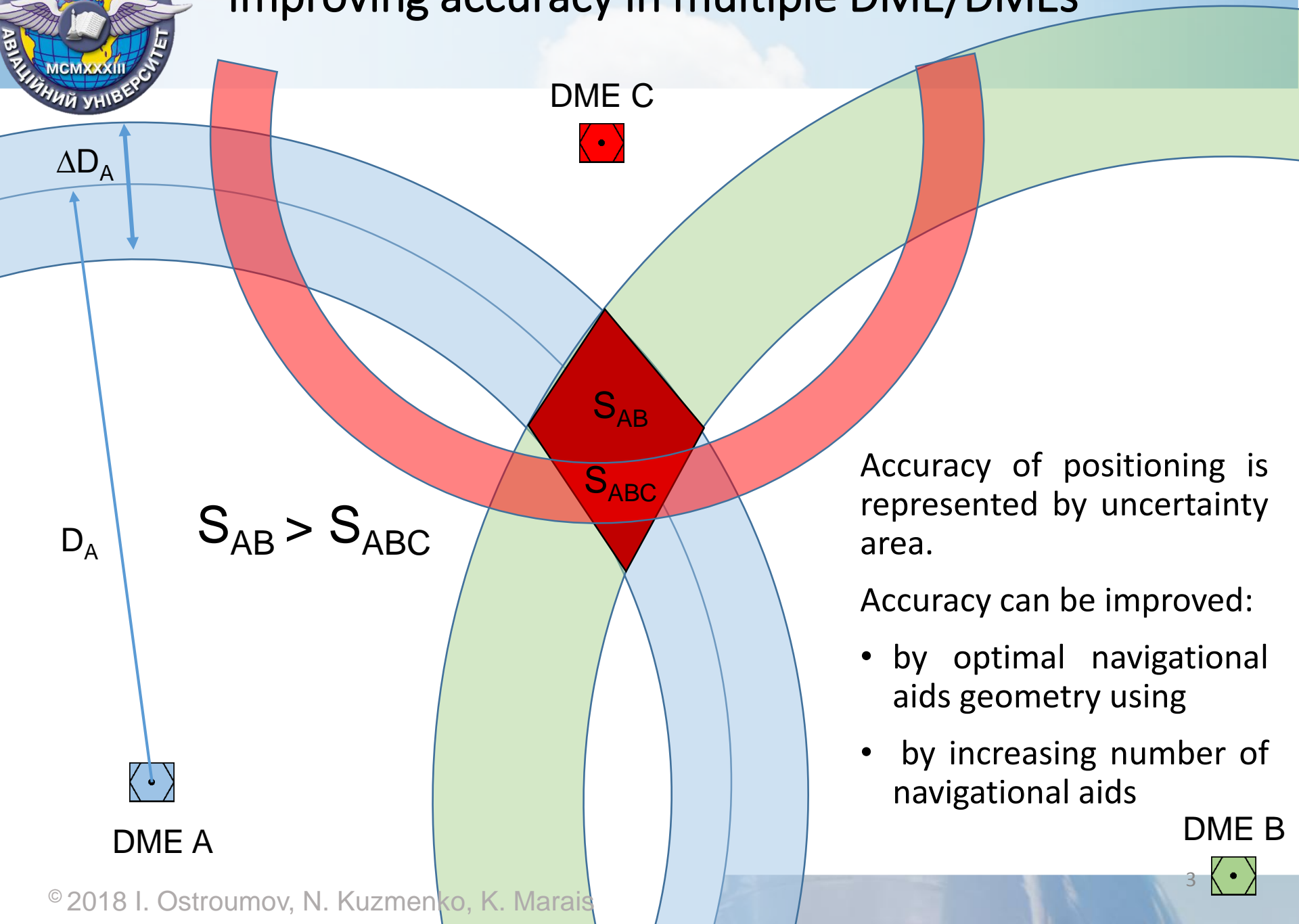
# Interchange of NAVAIDS data of on-board equipment



Only pair of DME/DME equipment is in use for positioning function.



# Improving accuracy in multiple DME/DMEs



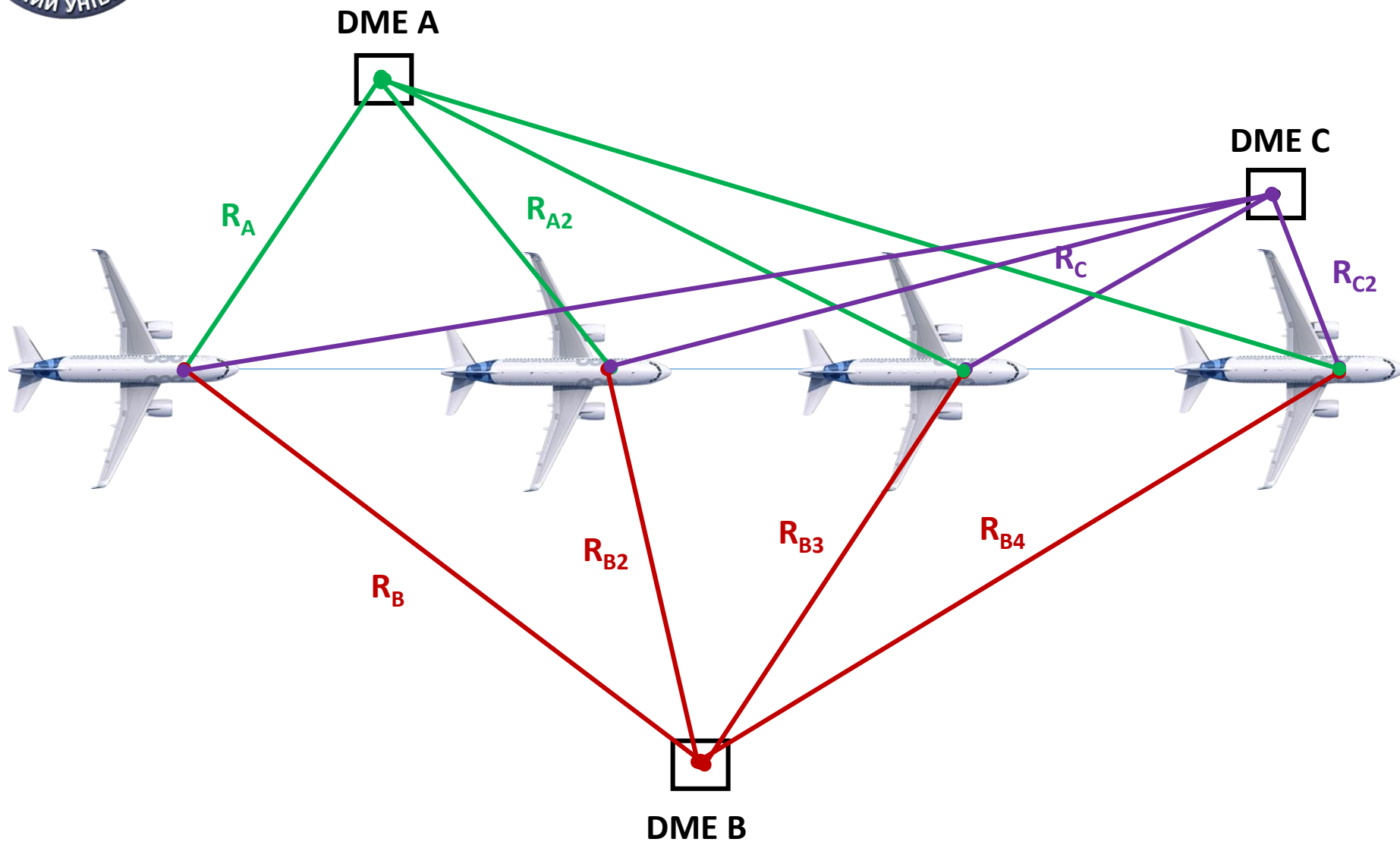
Accuracy of positioning is represented by uncertainty area.

Accuracy can be improved:

- by optimal navigational aids geometry using
- by increasing number of navigational aids



# DME/DME (one pair / multi pairs)





# DME/DME Performance

Root-mean-square deviation of radial error in horizontal plane for a pair of DME A and DME B

$$\sigma_P = \frac{\sqrt{\sigma_{DMEA}^2 + \sigma_{DMEB}^2}}{\sin(\alpha_{AB})}$$

$$\sigma_{DMEA,B}^2 = \sigma_{sis}^2 + \sigma_{air}^2$$

$\sigma_{sis}^2$  – Signal in space error;  $\sigma_{air}^2$  – Error of airborne interrogator  
 $\sigma_{sis} = 0.05 \text{ NM}$ ;  $\sigma_{air} = \max\{0.085 \text{ NM}; 0,125\% R\}$

In case of positioning by multiple DMEs

$$\sigma_0^2 (H_{DME}^T W_{DME} H_{DME})^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\ \sigma_{xy}^2 & \sigma_y^2 & \sigma_{yz}^2 \\ \sigma_{xz}^2 & \sigma_{yz}^2 & \sigma_z^2 \end{bmatrix}$$

where  $\sigma_0^2$  is root-mean-square error introduced by ground navigational aid;  
 $W_{DME}$  is a diagonal matrix of weight coefficients;  $H_{DME}$  is a matrix of partial derivatives by coordinates.

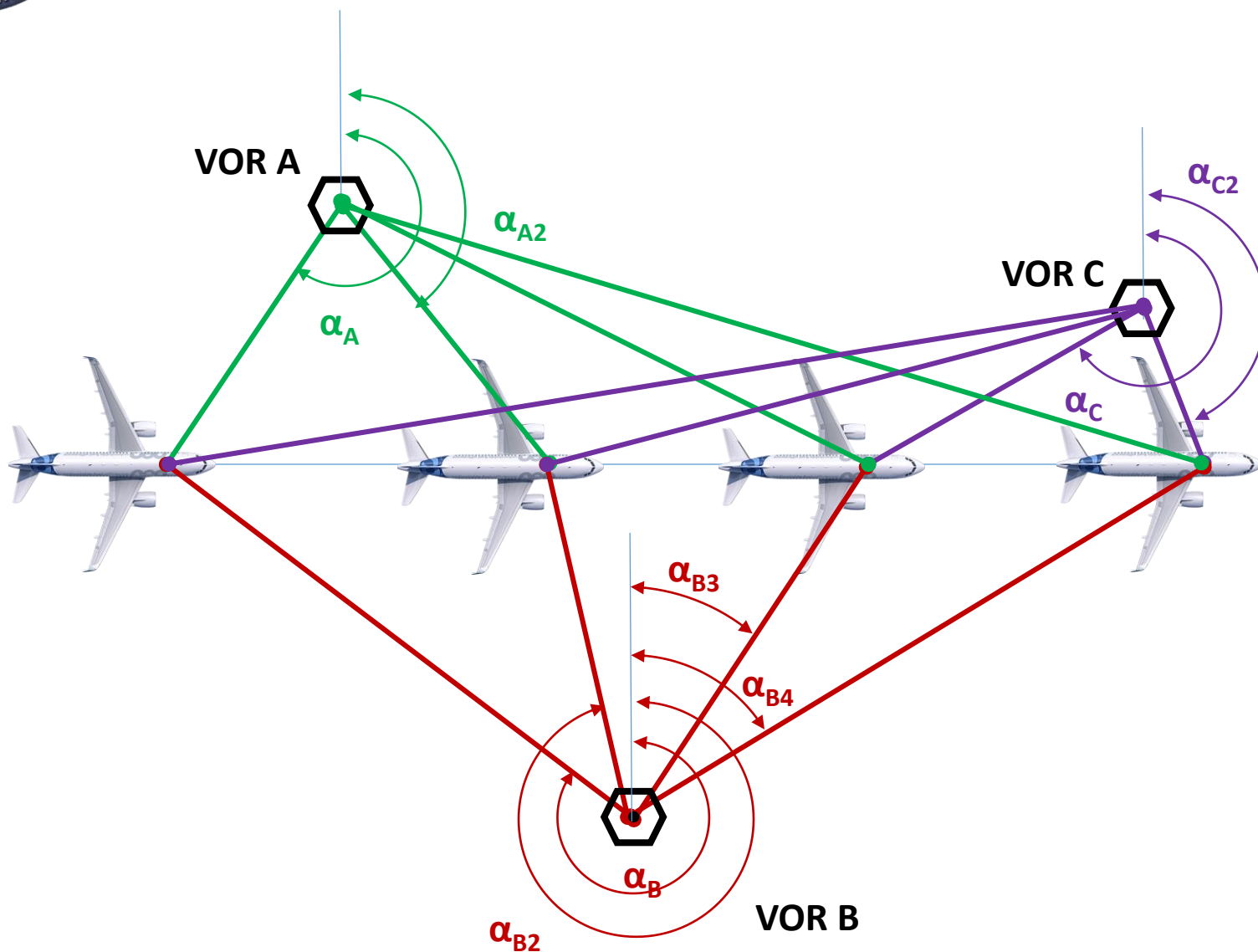
$$H_{DME} = \begin{bmatrix} \frac{x_0 - x_{DME1}}{d_{DME1}} & \frac{y_0 - y_{DME1}}{d_{DME1}} & \frac{z_0 - z_{DME1}}{d_{DME1}} \\ \frac{x_0 - x_{DME2}}{d_{DME2}} & \frac{y_0 - y_{DME2}}{d_{DME2}} & \frac{z_0 - z_{DME2}}{d_{DME2}} \\ \dots & \dots & \dots \\ \frac{x_0 - x_{DME_n}}{d_{DME_n}} & \frac{y_0 - y_{DME_n}}{d_{DME_n}} & \frac{z_0 - z_{DME_n}}{d_{DME_n}} \end{bmatrix} \quad W_{DME} = \begin{bmatrix} \frac{1}{\sigma_{DME1}^2} & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_{DME2}^2} & \dots & 0 \\ \dots & \dots & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_{DME_n}^2} \end{bmatrix}$$

Positioning error in horizontal plane with axes components can be estimated as follows:

$$\sigma_{p DME}^2 = \sigma_x^2 + \sigma_y^2$$



# VOR/VOR (one pair / multi pairs)





# VOR/VOR Performance

Root-mean-square deviation of radial error in horizontal plane for a pair of VOR A and VOR B

$$\sigma_p^2 = \frac{\sigma_\alpha^2}{\sin^2(\alpha)} (d_A^2 + d_B^2)$$

Where:

$$d_A = \frac{d \sin(\alpha_B)}{\sin(\alpha_A + \alpha_B)}$$

$$d_B = \frac{d \sin(\alpha_A)}{\sin(\alpha_A + \alpha_B)}$$

In case of positioning by multiple VORs

$$\sigma_{pVOR}^2 = \text{tr}(\sigma_0^2 (H_{VOR}^T W_{VOR} H_{VOR})^{-1}),$$

where  $W_{VOR}$  is a diagonal matrix of weight coefficients;  $H_{VOR}$  is the matrix of partial derivatives by coordinates.

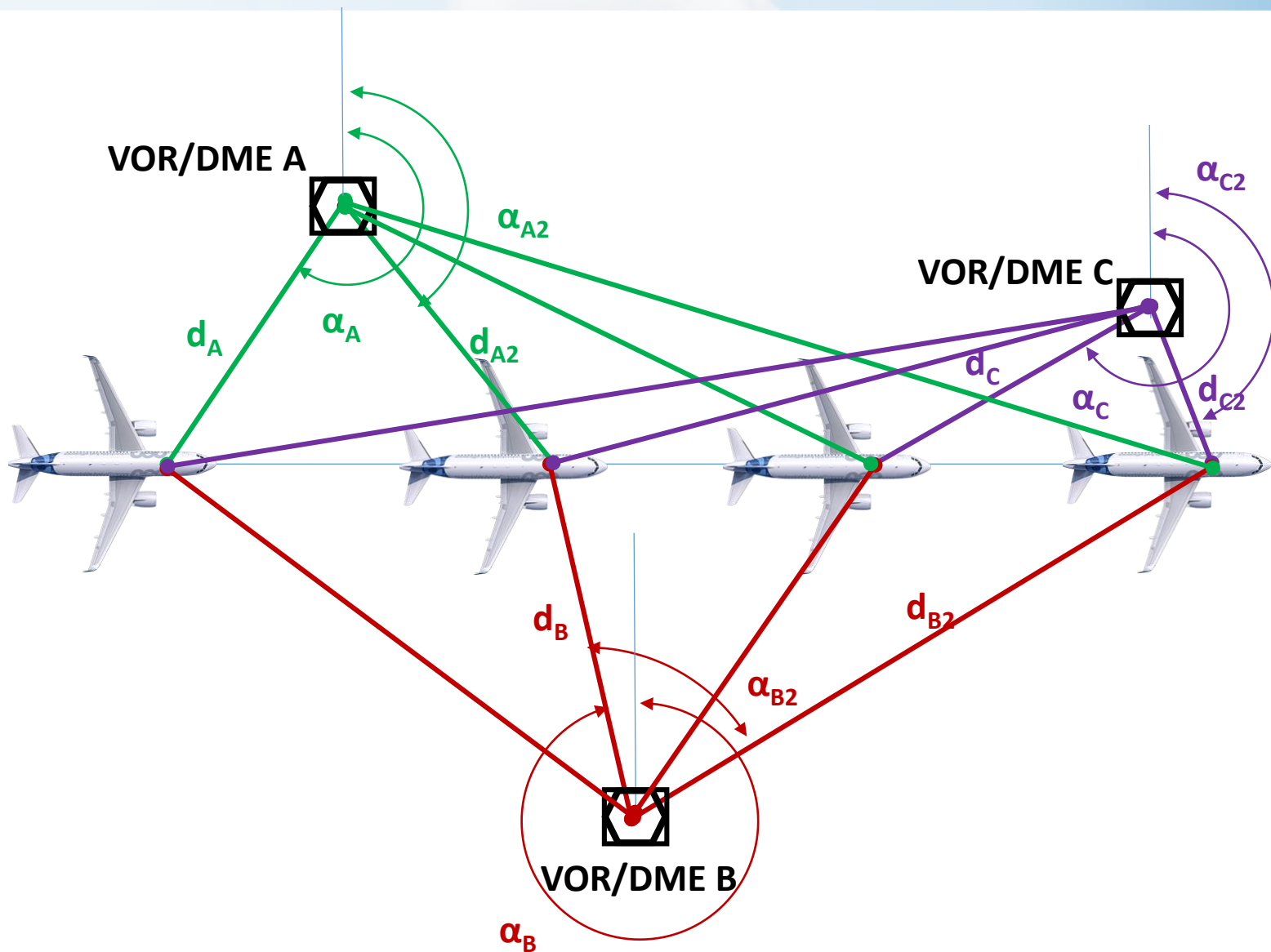
$$H_{VOR} = \begin{bmatrix} \frac{y_{VOR1} - y_0}{d_{VOR1}^2} & \frac{x_0 - x_{VOR1}}{d_{VOR1}^2} \\ \frac{y_{VOR2} - y_0}{d_{VOR2}^2} & \frac{x_0 - x_{VOR2}}{d_{VOR2}^2} \\ \dots & \dots \\ \frac{y_{VORn} - y_0}{d_{VORn}^2} & \frac{x_0 - x_{VORn}}{d_{VORn}^2} \end{bmatrix}$$

$$H_{VOR} = \begin{bmatrix} \frac{\sin(\alpha_{VOR1})}{d_{VOR1}} & -\frac{\cos(\alpha_{VOR1})}{d_{VOR1}} \\ \frac{\sin(\alpha_{VOR2})}{d_{VOR2}} & -\frac{\cos(\alpha_{VOR2})}{d_{VOR2}} \\ \dots & \dots \\ \frac{\sin(\alpha_{VORn})}{d_{VORn}} & -\frac{\cos(\alpha_{VORn})}{d_{VORn}} \end{bmatrix}$$

$$W_{VOR} = \begin{bmatrix} \frac{1}{\sigma_{VOR1}^2} & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_{VOR2}^2} & \dots & 0 \\ \dots & \dots & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_{VORn}^2} \end{bmatrix}$$



# VOR/DME (one pair / multi pairs)





# VOR/DME Performance

Root-mean-square deviation of radial error in horizontal plane for a pair of VOR A and VOR B

$$\sigma_p^2 = \sigma_{DME}^2 + D^2 \sigma_{VOR}^2$$

In case of positioning by multiple VORs system of navigation equations

$$X = (A^T A)^{-1} A^T B$$

where

$X$  is the matrix of aircraft coordinates;

$A$  is a binary matrix that groups elements of navigation equation by  $x$  and  $y$  components;

$B$  is a matrix of navigation equations.

$$X = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ \dots & \dots \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ \dots & \dots \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} x_1 + d_1 \cos(\alpha_1) \\ x_2 + d_2 \cos(\alpha_2) \\ \dots \\ x_n + d_n \cos(\alpha_n) \\ y_1 + d_1 \sin(\alpha_1) \\ y_2 + d_2 \sin(\alpha_2) \\ \dots \\ y_n + d_n \sin(\alpha_n) \end{bmatrix}$$

$$\sigma_{pVOR/DME}^2 = \text{tr}(\sigma_0^2 (H_{VOR/DME}^T W_{VOR/DME} H_{VOR/DME})^{-1})$$

where

$W_{VOR/DME}$  is a matrix of weight for navigation data measurements;

$H_{VOR/DME}$  is a matrix of partial derivatives by coordinates:

$$H_{VOR/DME} = \begin{bmatrix} H_{DME} \\ H_{VOR} \end{bmatrix}$$



# Verifications

“MSI263” Kyiv (UKBB) / Lviv (UKLL) , May 29, 2018

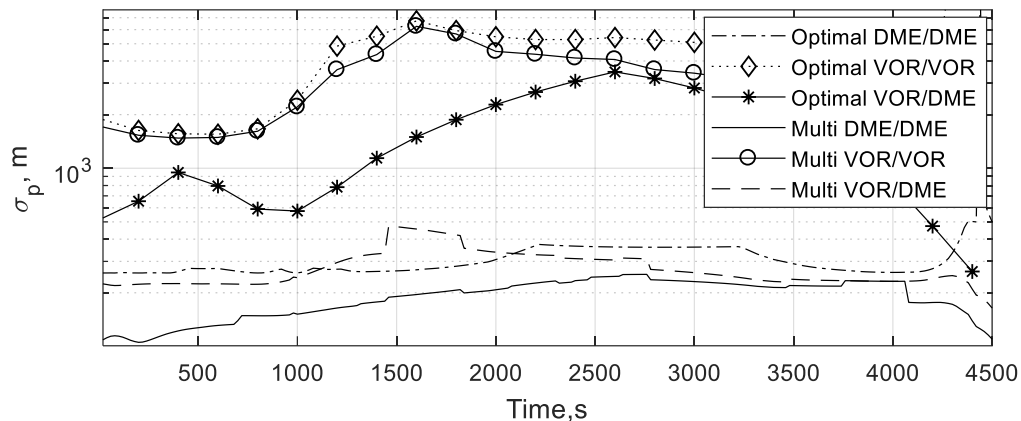
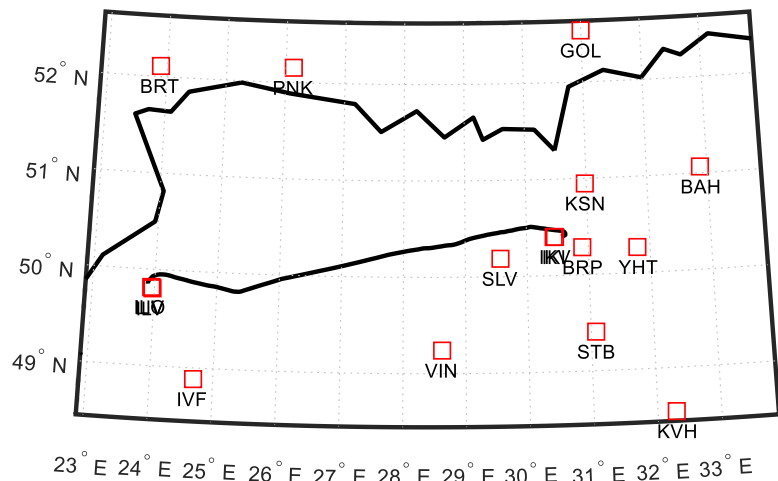


Fig. 10.3. Accuracy estimation of different positioning methods

Fig. 10.1. Representation of "MSI263" flight track

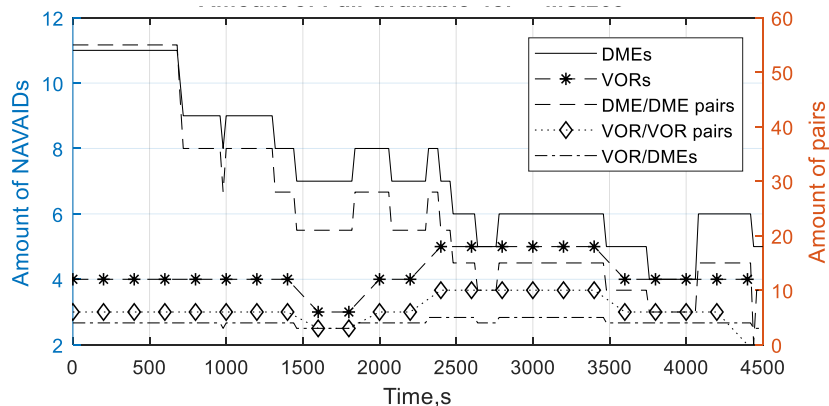


Fig. 10.2. The total amount of available DMEs, VORs and their pair combinations

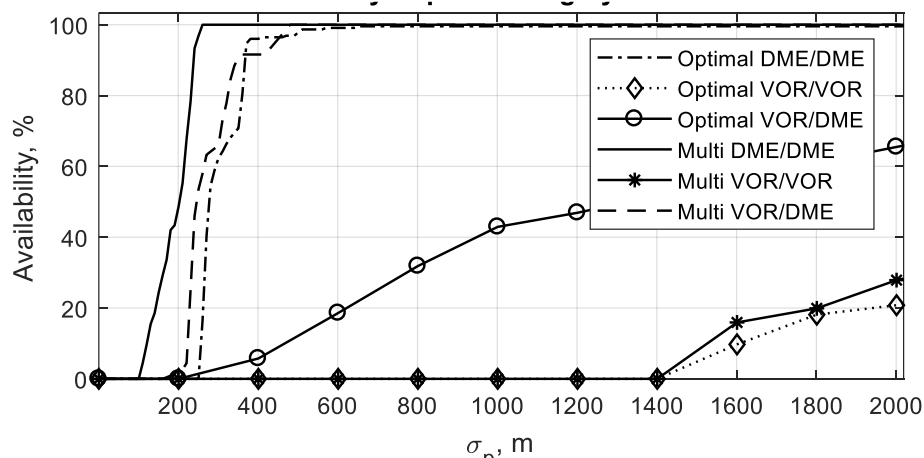


Fig. 10.4. Availability of positioning by  $\sigma_p$  for different methods

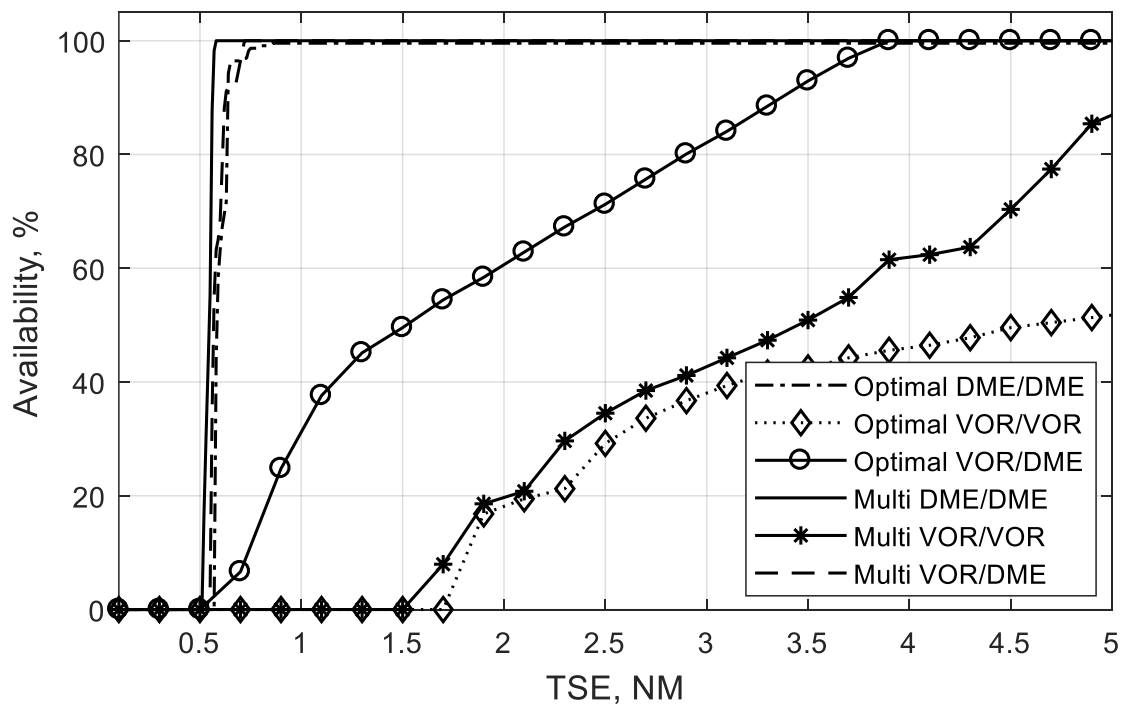


# Total System Error

*TSE includes Navigation System Error (NSE), Flight Technical Error (FTE), and Path Definition Error (PDE):*

$$TSE^2 = NSE^2 + FTE^2 + PDE^2.$$

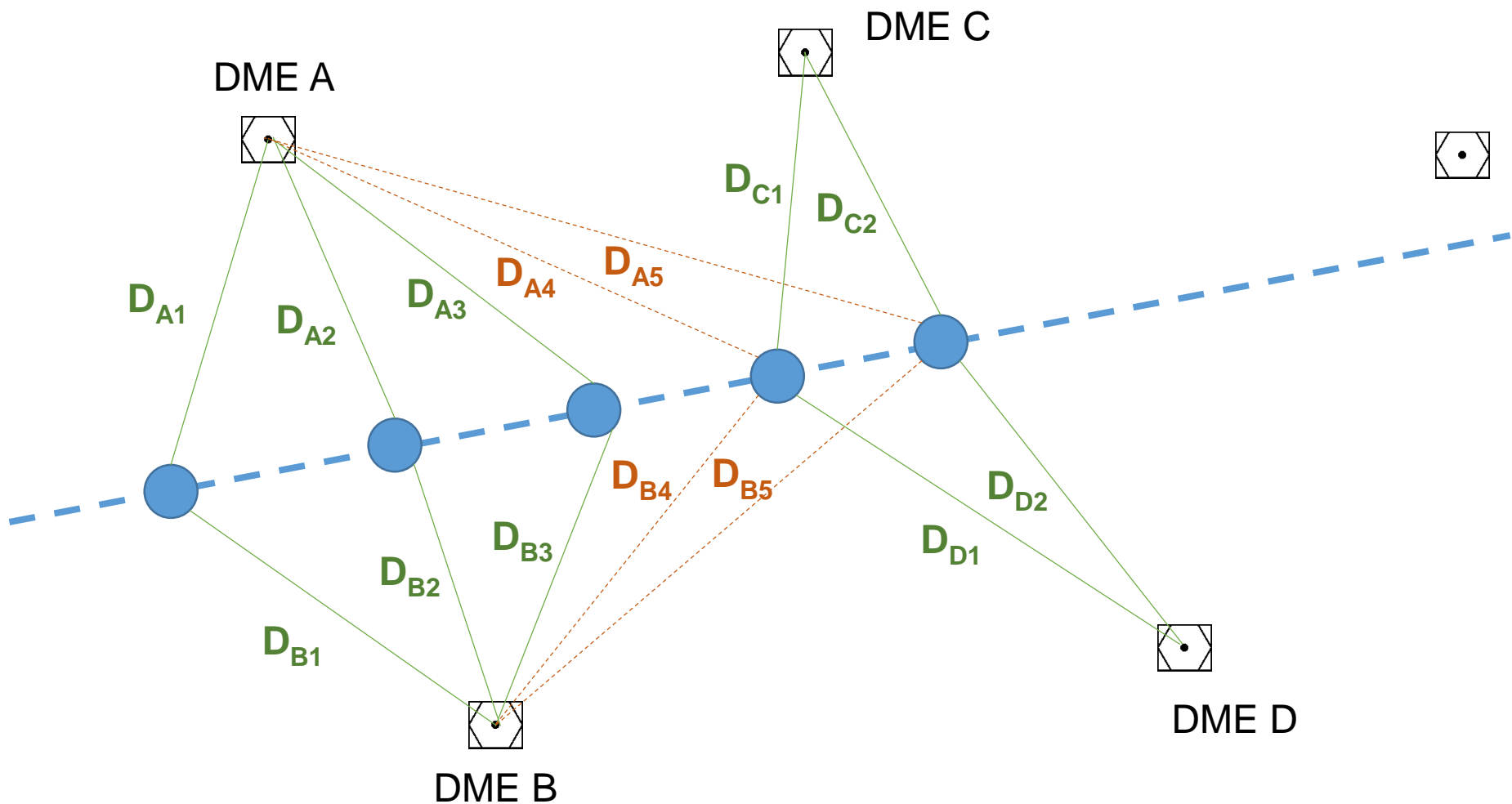
$$TSE^2 = 4 \sigma_p^2 + FTE^2$$



*Fig. 11.1. Availability of positioning by TSE for different methods*

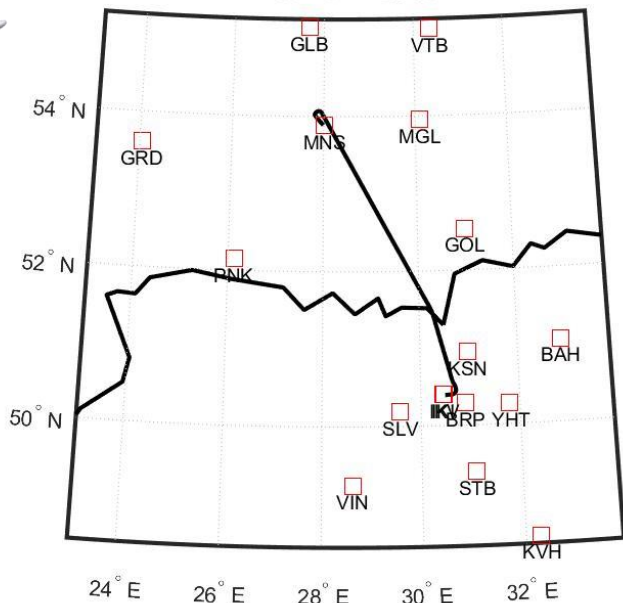


# Multi DME/DMEs positioning

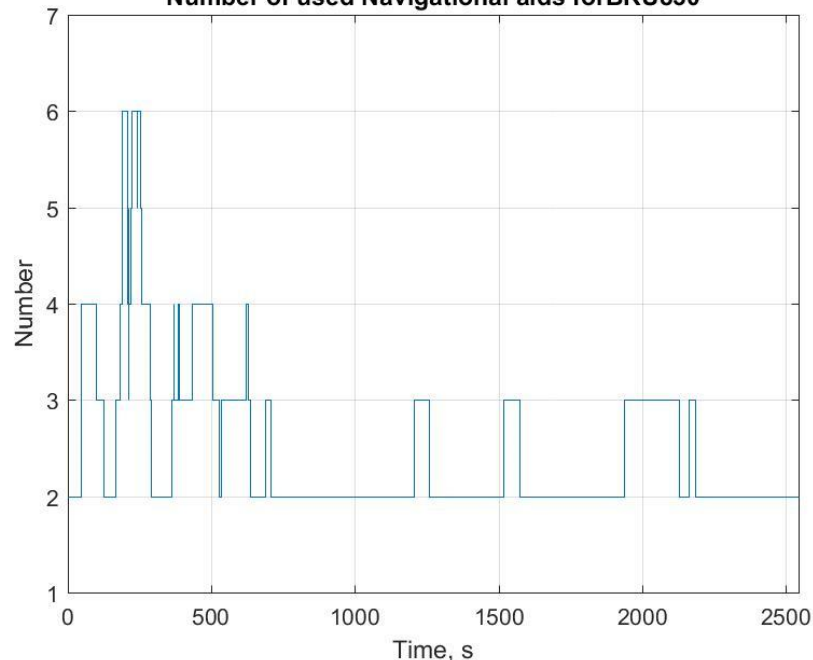




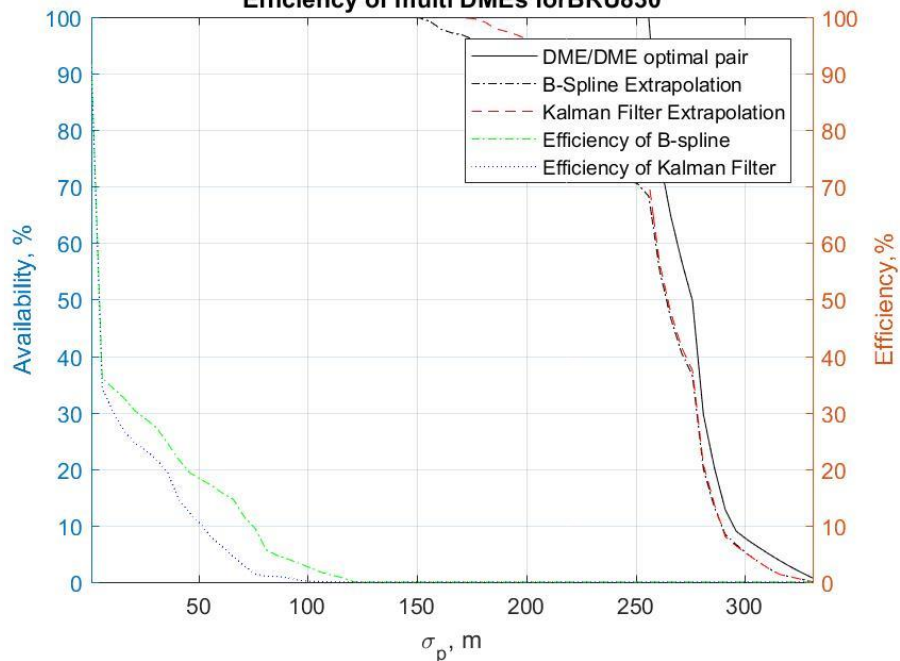
Track of BRU830



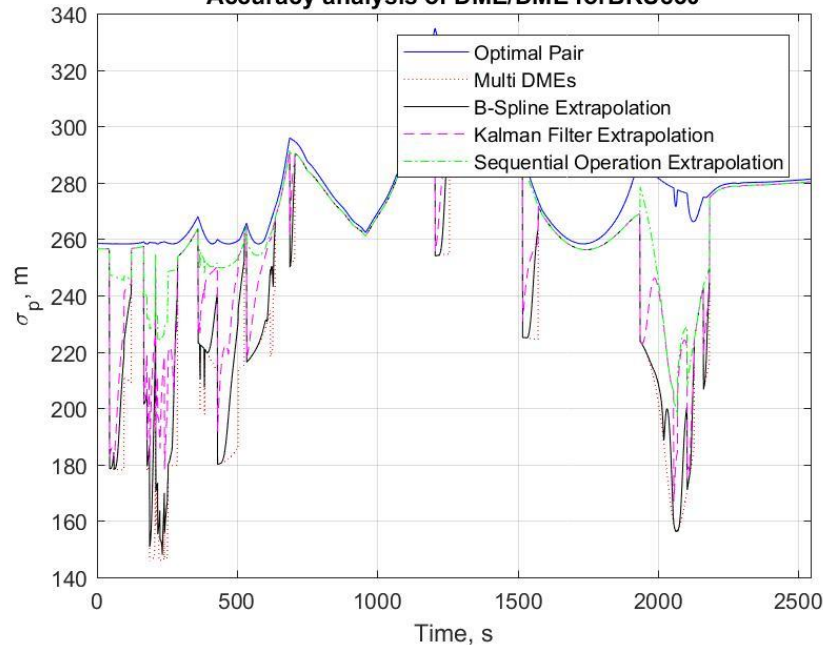
Number of used Navigational aids for BRU830



Efficiency of multi DMEs for BRU830



Accuracy analysis of DME/DME for BRU830





# Conclusions

Obtained results of the comparison of different positioning methods and accuracy of their performance show a significant increase in accuracy and availability for positioning methods by multiple navigational aids. In particular, results of positioning by multiple DME/DMEs and VOR/DMEs for investigated flight trajectory indicate a significant increase in accuracy in comparison with other methods and even are better than positioning by optimal DME/DME pair, which undoubtedly proves the importance of further research towards the development of positioning systems by multiple navigational aids.