National Aviation University of Ukraine
Air Navigation Systems Department

An area navigation (RNAV) system performance monitoring and alerting

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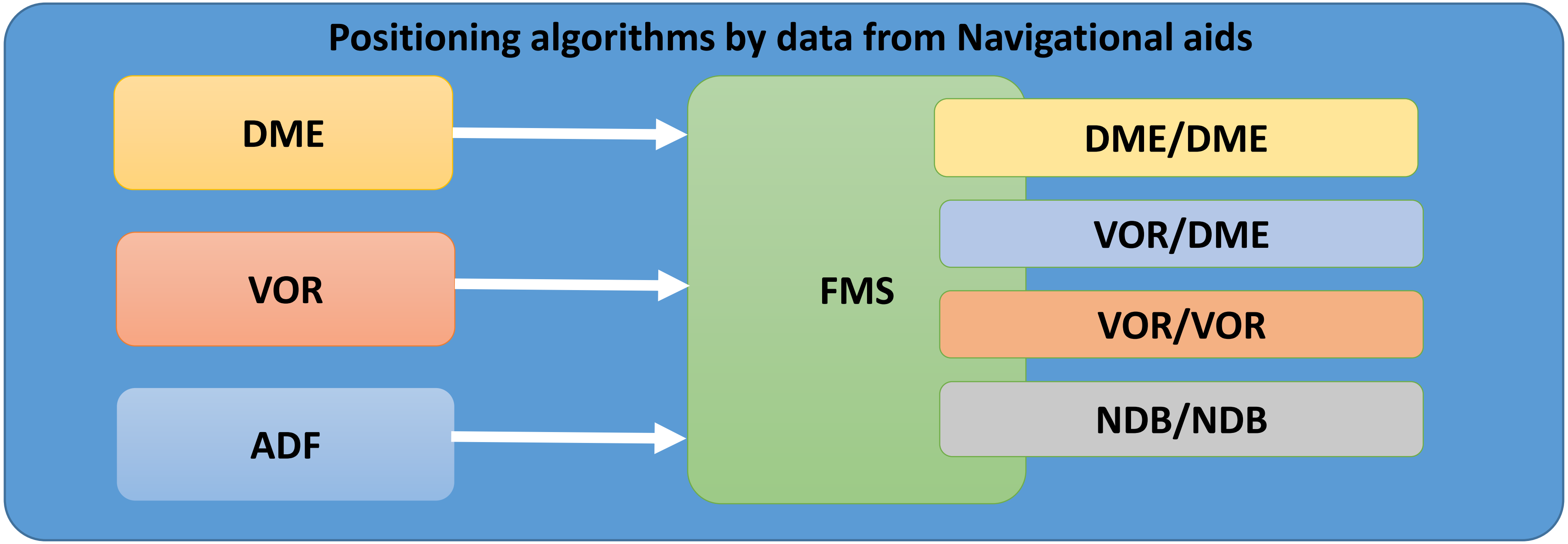
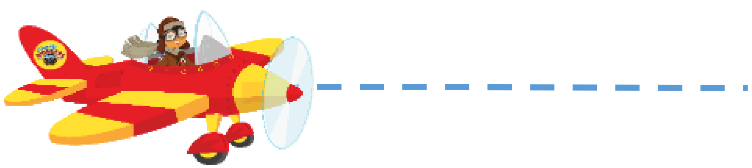
An area navigation (RNAV) system

Global navigational satellite system (GNSS)

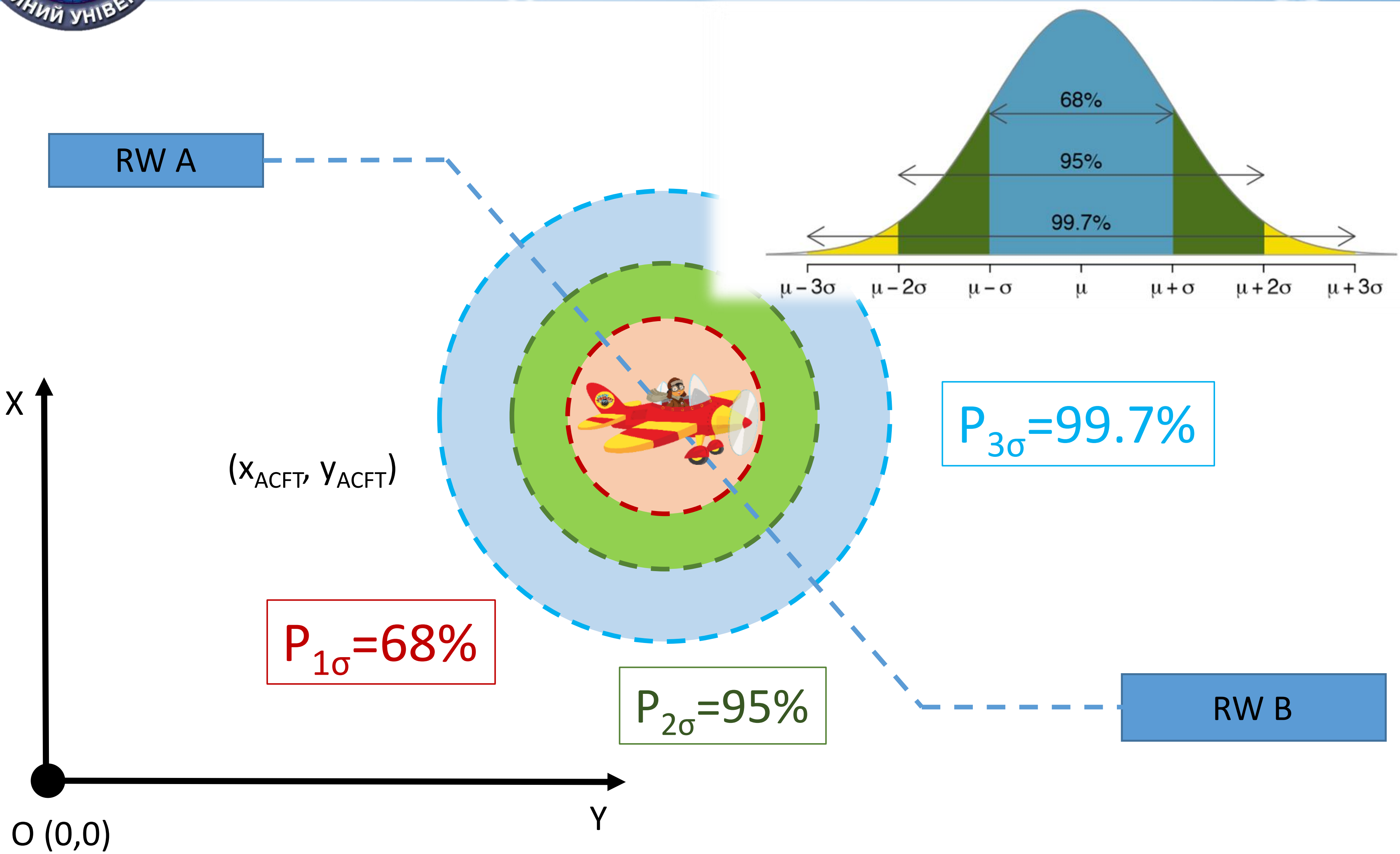
Inertial Navigation System (INS)



An area navigation (RNAV) system is any system that allows the aircraft to be navigated to the required level of accuracy without the requirement to fly directly over ground based facilities.



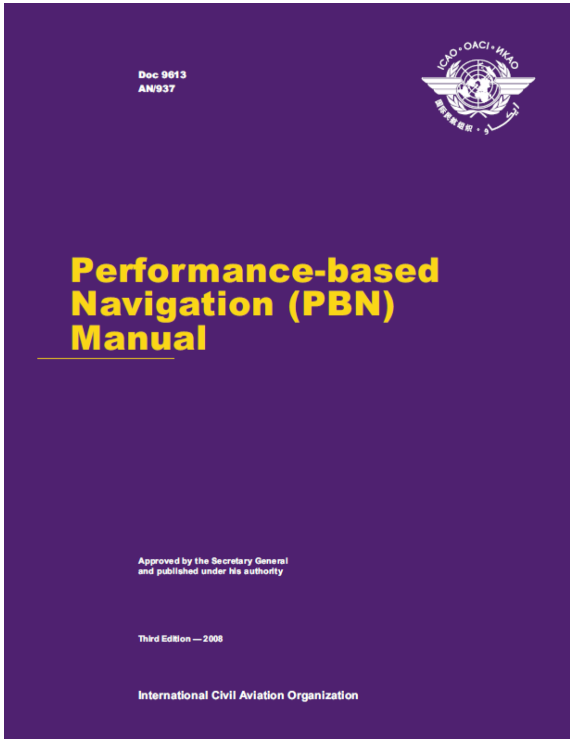
Errors of positioning



Performance-based navigation(PBN) **is** Area navigation based on performance requirements for aircraft operating along an ATS route, on an instrument approach procedure or in a designated airspace. Performance requirements are expressed in navigation specifications in terms of :

- accuracy,
- integrity,
- continuity,
- availability,
- functionality needed

for proposed operation in the context of a particular airspace concept.



ICAO DOC 9613

Navigation Specifications

Navigation Specifications

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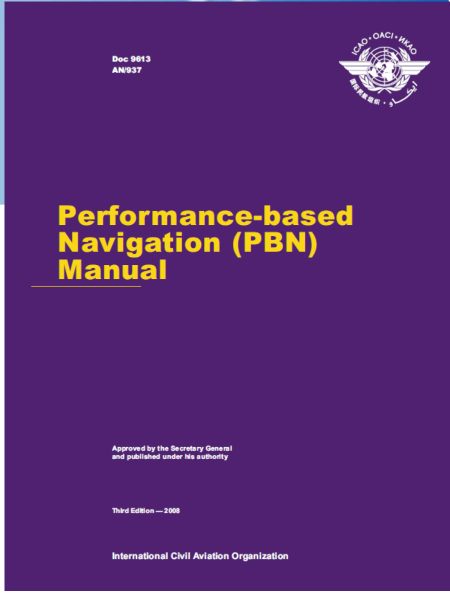


RNAV SPECIFICATIONS

Designation
RNAV 10
For oceanic and remote continental navigation applications

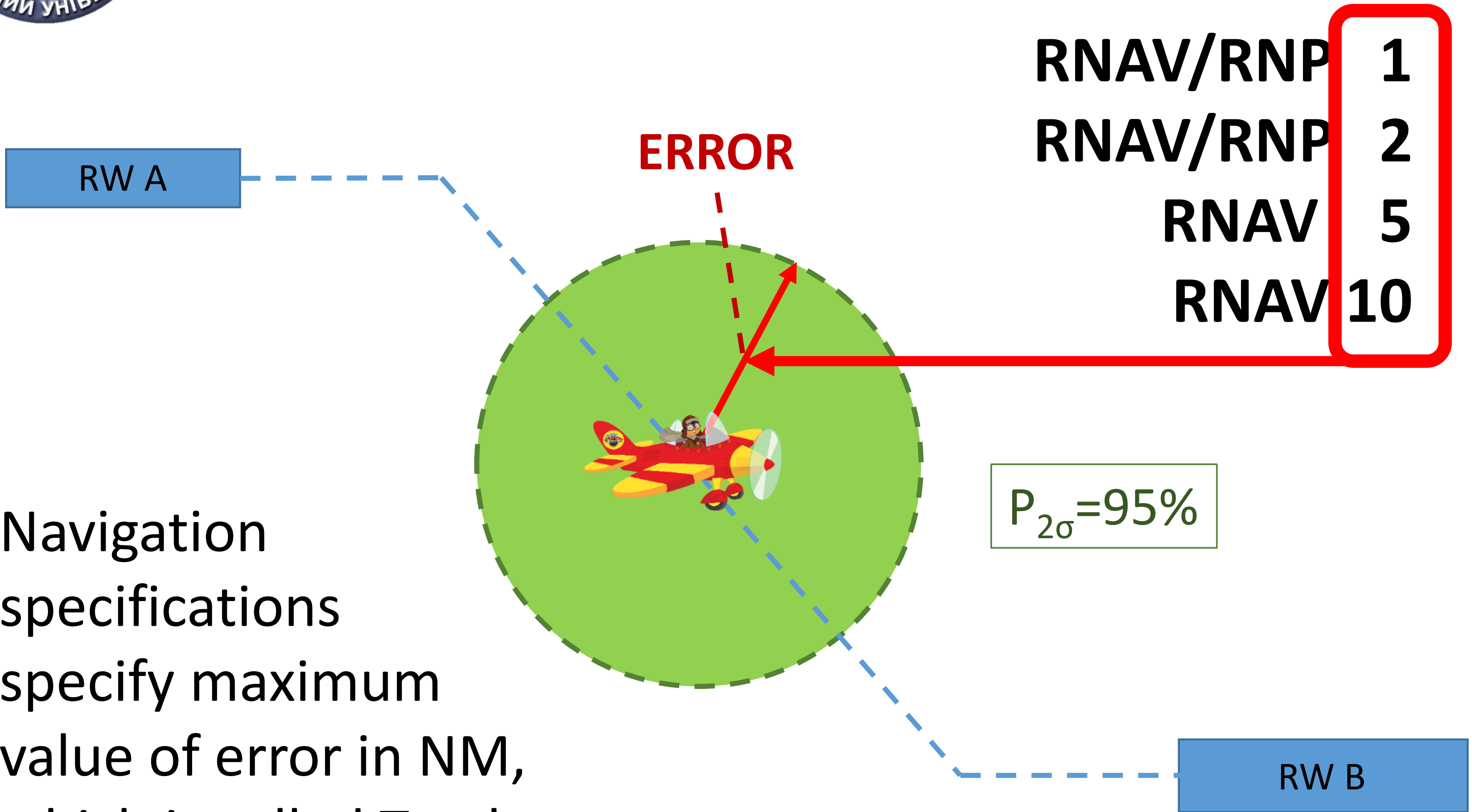
Designation
RNAV 5
RNAV 2
RNAV 1
For en-route and terminal navigation applications

RNP SPECIFICATIONS

Designation
RNP 4
For oceanic and remote continental navigation applications

Designation
RNP 2
Basic-RNP 1
Advanced-RNP 1
RNP APCH
RNP AR APCH
for various phases of flight

Designation
RNP with additional requirements to be determined (e.g. 3D, 4D)



Navigation specifications specify maximum value of error in NM, which is called Total System Error (TSE)

$$TSE^2=NSE^2+FTE^2+PDE^2$$

NSE
Navigation system error

FTE
Flight technical error

PDE - Path definition error (PDE=0)

Monitoring and Alerting

- Nowadays comparative approach (simple comparison current value of TSE with RNP specification value). Does not take into account statistical data.
- We propose probabilistic approach for performance monitoring and alerting.
- Each of system states can be represented as a certain hypothesis A $(A_1, A_2, \ldots, A_N)$.
- prior probabilities of these hypotheses are equally probable for the query:

$$P(A_1)=P(A_2)=\ldots=P(A_N)=N^{-1}$$

- Bayesian formula allows to estimate a posterior probability of system state $P(A/b)$, which reflects the probability that A hypothesis contains b information:

$$P(A_i/b_j)=\frac{P(A_i)P(b_j/A_i)}{\sum_{i=1}^{N}P(A_i)P(b_j/A_i)}$$

where $P(A_1)$ is a prior probabilities of system states; $P(b/A)$ is a probability of b under the condition of A state presence.

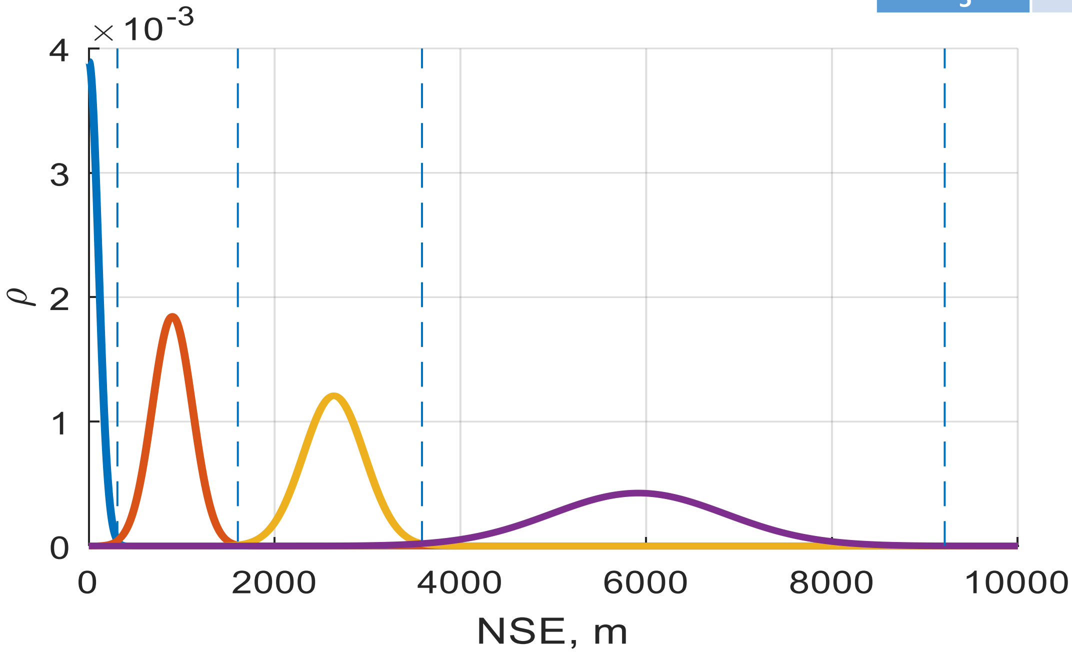
- Bayesian recognition criterion is based on the comparison of a posterior probabilities of the presence of b performance value within A state. After performance estimation, these probabilities are estimated by the formula (2). When comparing all values of $P(A/b)$, a system state refers to a class to which this value is maximum, providing a minimum probability error.

Model of the state classifier by NSE value

$$NSE^2 = TSE^2 - FTE^2$$

Each system state can be characterized by specific Conditional Probability Density Functions (CPDF) ρ(σ)

System state	System states in terms of RNP	Boundary of state class	Formula for σ_k calculation
A_1	RNP 0.3	NSE_1	$\sigma_1=NSE_1/3$
A_2	RNP 1	NSE_2	$\sigma_2=(NSE_2-NSE_1)/6$
A_3	RNP 2	NSE_3	$\sigma_3=(NSE_3-NSE_2)/6$
A_4	RNP 5	NSE_4	$\sigma_4=(NSE_4-NSE_3)/6$
A_5	out of RNP	not defined	$\sigma_5=\sigma_4$



Mathematical expectation of the first CPDF is assumed to be equal to zero (*$\mu_1=0$*), other ones are selected as follows:

$$\rho_i = \frac{1}{\sigma_i\sqrt{2\pi}} exp\left(-\frac{(NSE_i-\mu_i)}{2\sigma_i^2}\right)$$

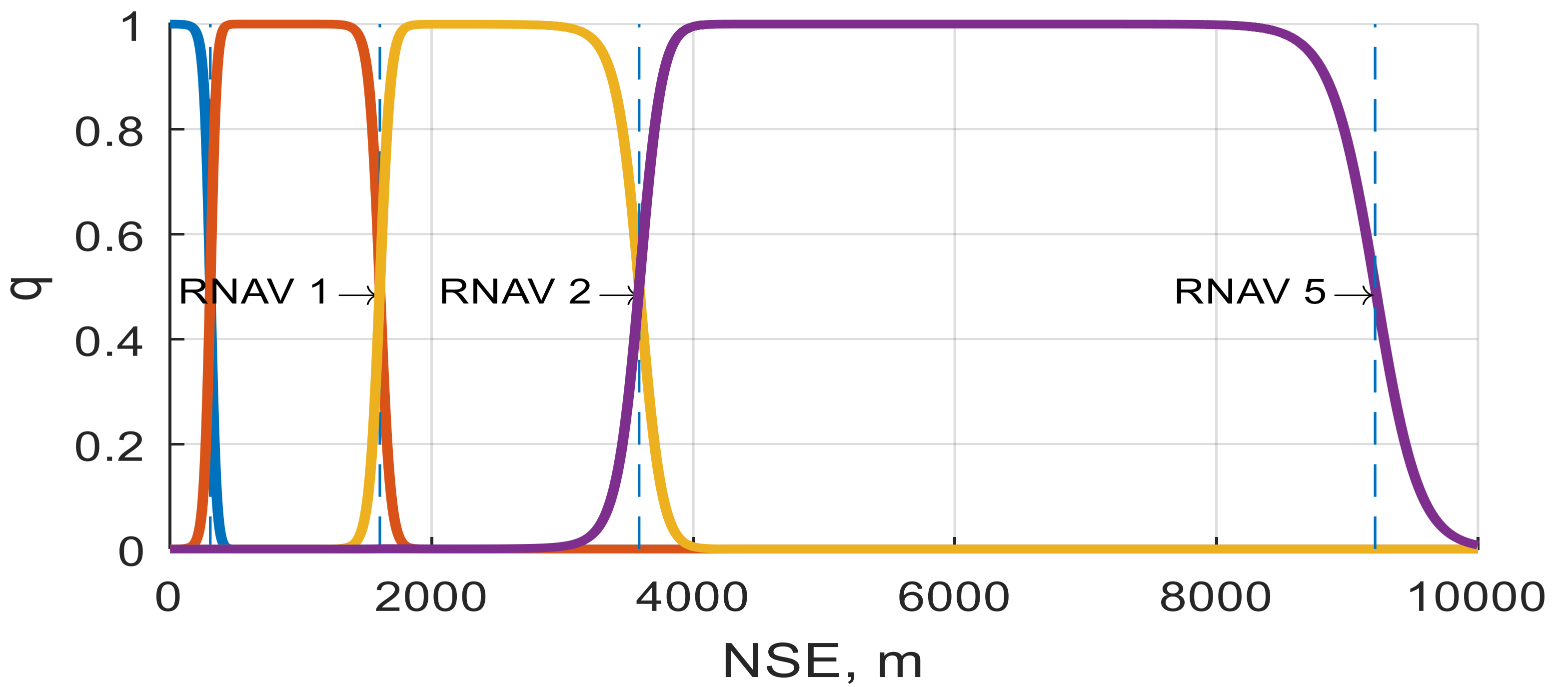
$$\mu_{i+1} = NSE_i - \sigma_{i+1}\sqrt{2\,ln\left(\rho_i\sigma_{i+1}\sqrt{2\pi}\right)}$$

Model of the state classifier by NSE value

Classifier model by a posterior probability can be written as follows:

$$q_k(x) = \frac{p_k \rho_k(x)}{\sum_{j=1}^{N} p_j \rho_j(x)},$$

Decision in favor of H_k hypothesis is taken according to condition: $q_k(x)=max(q_j(x)), j=1,N$.



Model of state classification according to longitudinal and lateral deviation

Bayesian formula with multiparameteric Gaussian distribution can be used as a classifier model

$$q_k\left(x_{lat}, x_{long}\right) = \frac{p_k \rho_k\left(x_{lat}, x_{long}\right)}{\sum_{j=1}^{N} p_j \rho_j\left(x_{lat}, x_{long}\right)}$$

In this case, each state of the system is characterized by a certain bivariate Gaussian CPDF:

$$\rho_k\left(x\right) = \frac{1}{(2\pi)^{K/2}\left|B_k\right|^{-1/2}} exp\left[-\frac{1}{2}\left(B_k^{-1}(x-M_k)^T \times (x-M_k)\right)\right]$$

where $M_k=(\mu_{lat}, \mu_{lon})$ is a vector of mathematical expectations for each parameter, estimated by formula (4); B_k is a matrix of mean-square deviations:

$$B_k = \begin{vmatrix} \sigma_{lat}^2 & 0 \\ 0 & \sigma_{lon}^2 \end{vmatrix}$$

Since the boundary values of lateral and longitudinal deviation are the same, than it has the following form:

$$B_k=diag([\sigma^2_k, \sigma^2_k]).$$

Verification

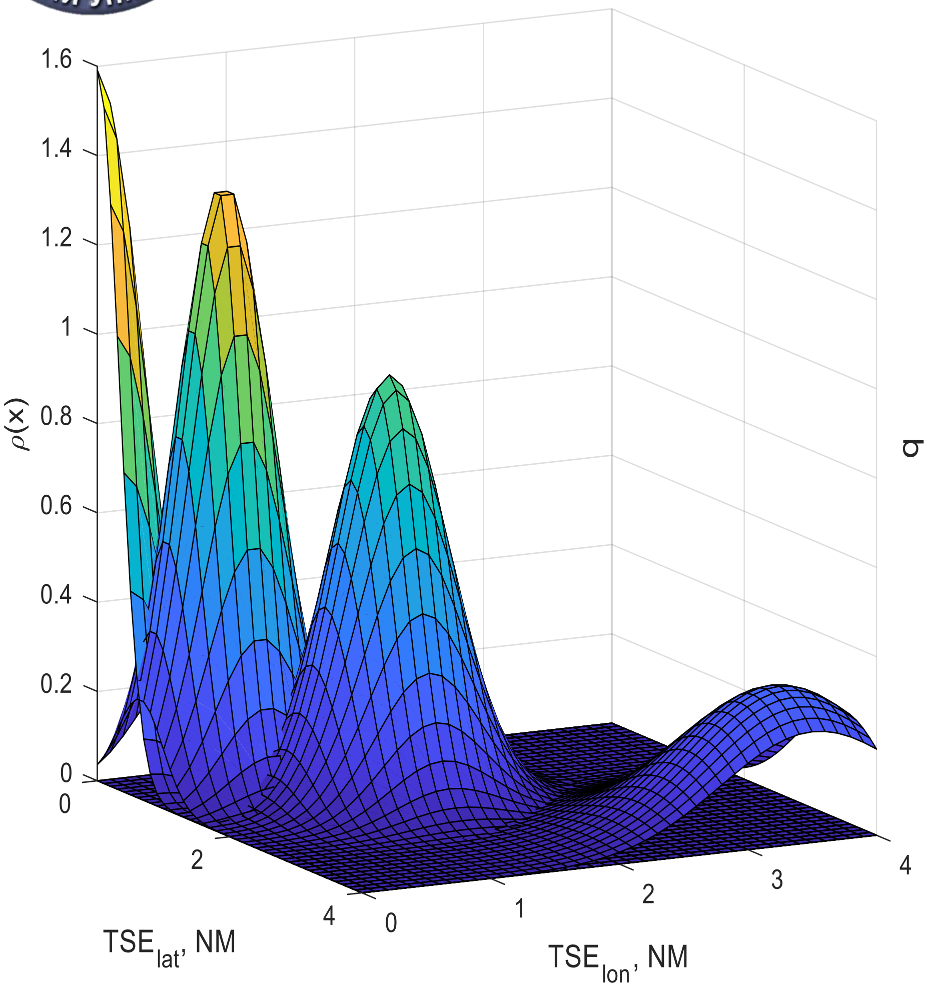


Fig. 12. 1. Bivariate CPDFs

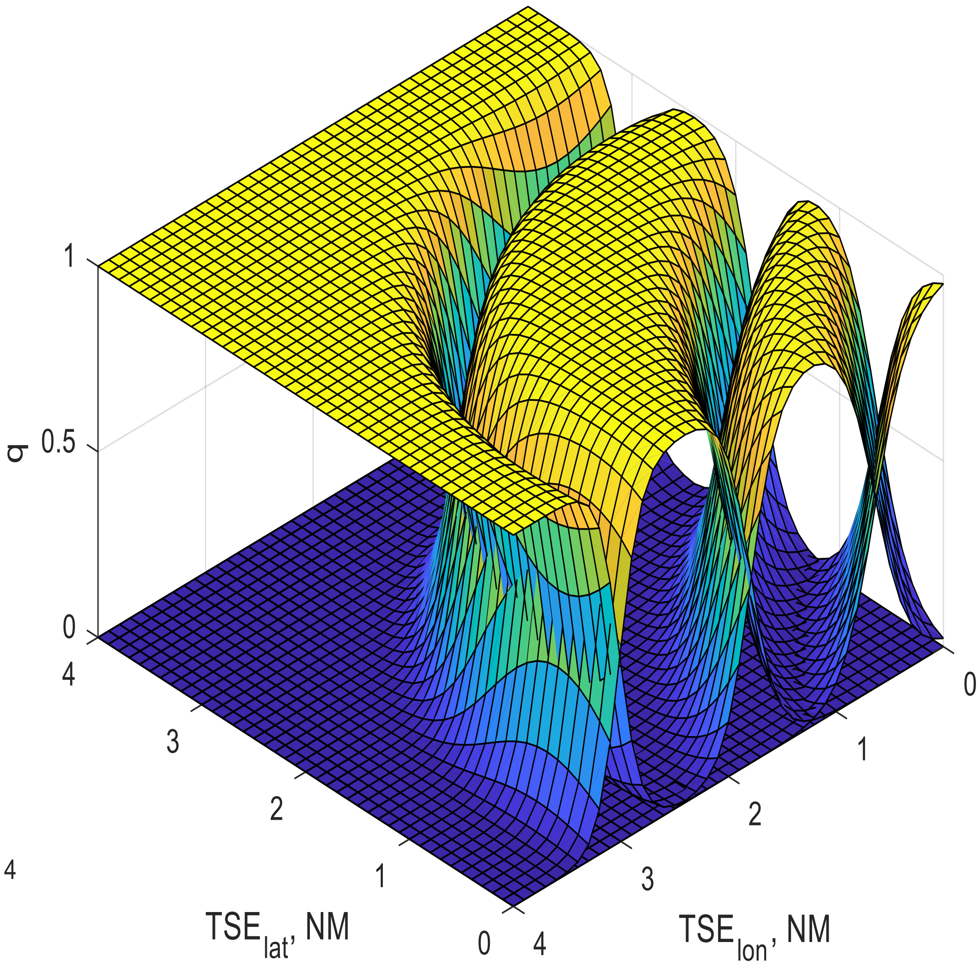
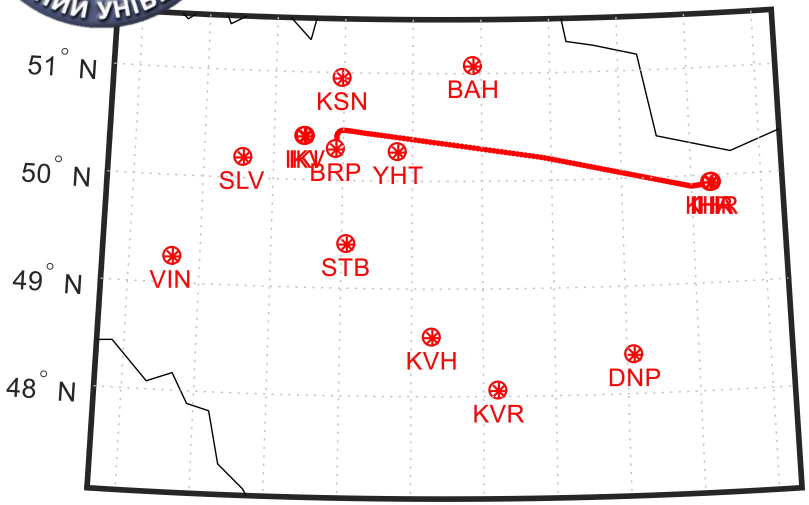


Fig. 12. 2. Posterior probabilities of the system state monitoring.

Verification (AUI 25, «UKBB» - «UKHH», May 14, 2018)



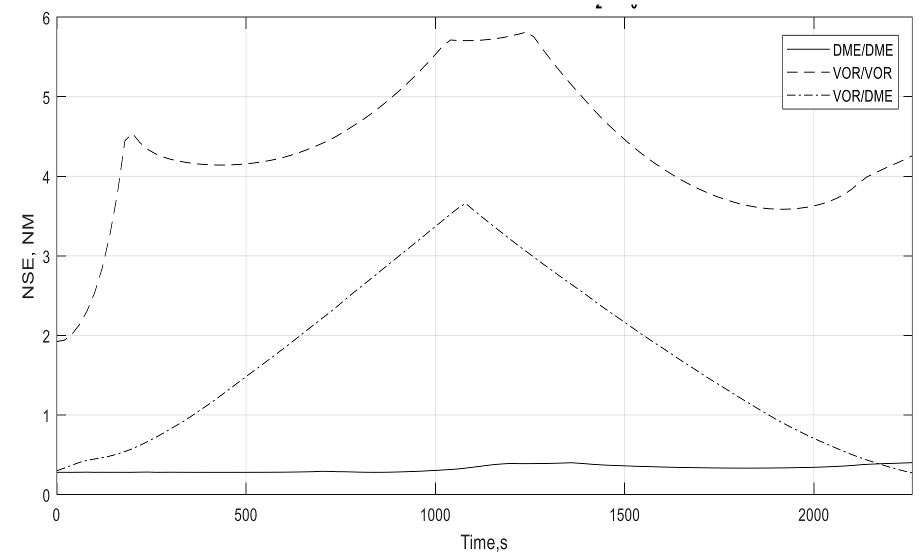


Fig. 13. 2. NSE

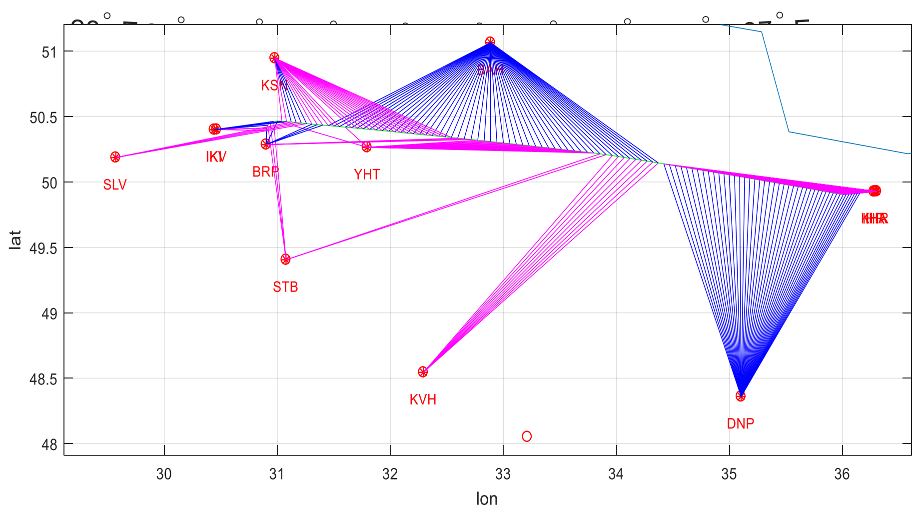
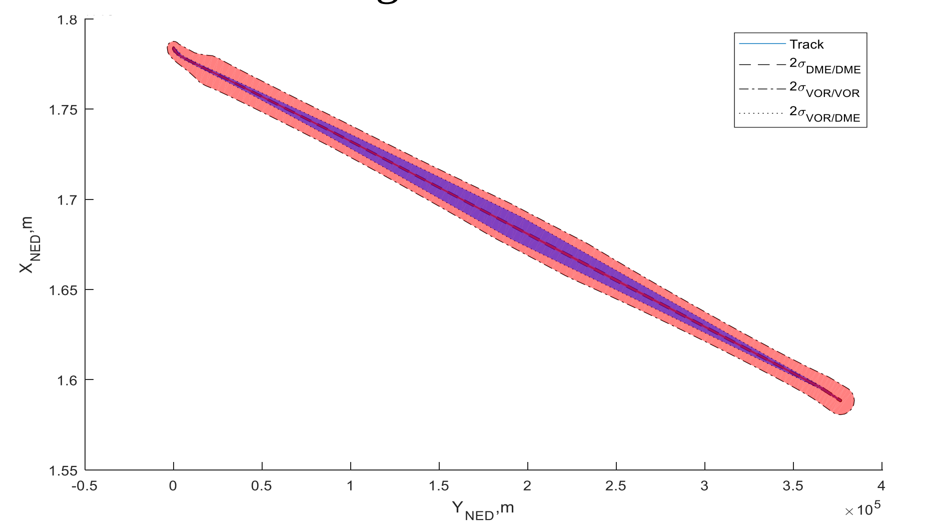


Fig. 13. 1. DME/DME optimal pair



Conclusions

Usage of a probabilistic approach for decision making about determining of the navigation system compliance with the necessary safety level associated to RNP application while navigating an aircraft allows to obtain a solution with minimal risk and simultaneously assess the probability of a solution provided with minimum hardware costs. A simple mathematical expression of posterior probability estimation allows to apply a probabilistic approach in small iteration steps.

An important stage in the determining a compliance with RNP requirements is a choice of CPDF parameters such as μ and σ. Established formula for μ and σ values from table allows to fit CPDFs to achieve precise values for monitoring navigation system state.

Proposed model of state classification according to longitudinal and lateral deviation allows to estimate the navigation system state according to accuracy characteristics in horizontal plane of aircraft body coordinate system that is going to be compatible with future RNP specifications. Obtained results are important for further implementation in algorithms of FMS as a mathematical tool for monitoring the compliance of the navigation system with RNP specifications.